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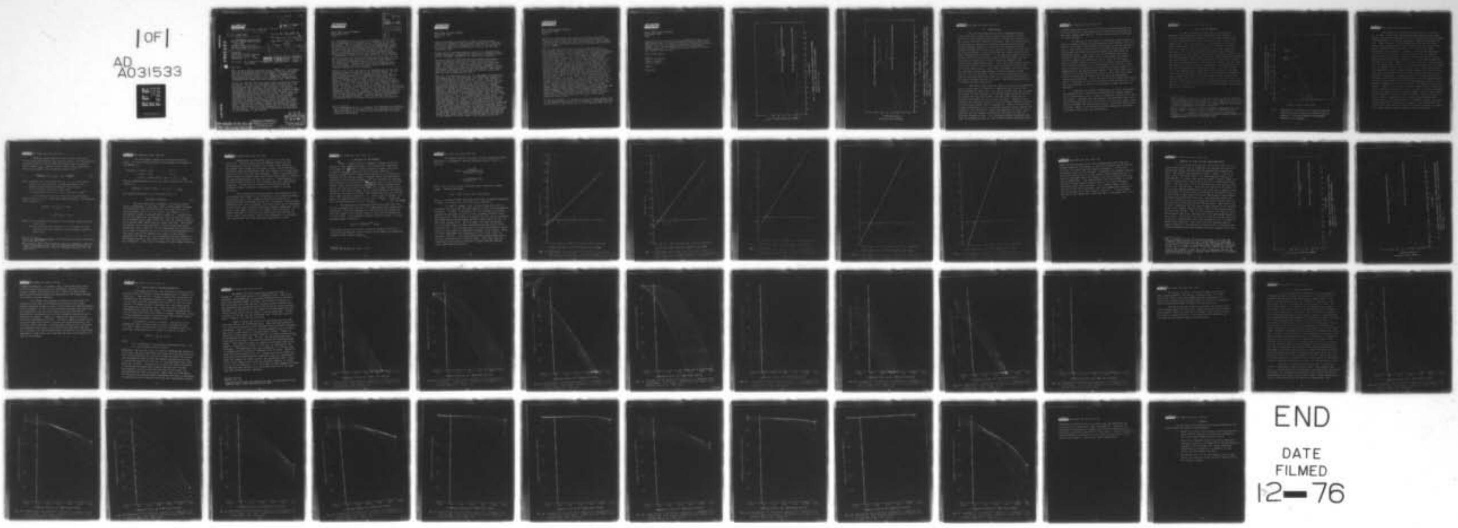

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Second Quarterly Progress Report from 1 June 1969 - 1 Sep 1969

9 Quarterly Progress Report Number 2, 1 June 1969 - 1 September 1969.

Commander
Naval Ship Systems Command
Department of the Navy
Washington, D. C. 20360

Attention: Mr. Kenneth Buske
Code OOV1C

Dear Sir:

This second quarterly progress report, submitted in accordance with the data requirements of the subject contract, describes the work accomplished from 1 June 1969 to 1 September 1969. The purpose of this contract is to investigate means of reducing requirements placed on the computer by the sequential likelihood ratio (SLR) processor developed under NObsr-93352.

During this period effort has been directed toward the following tasks. (1) The log likelihood ratio associated with SLR processor has been revised. This revision is based on a restatement of the hypotheses being tested. The results are satisfying in the sense that not only was the computer loading significantly reduced in comparison to the baseline case, but the apparent track intensity was increased. (2) A model was developed which may be used to calculate display marking probabilities after N pings. This model eliminates the necessity of establishing marking statistics through lengthy numerical experiments; and, consequently, parametric studies with the SLR can be conducted cheaply and quickly with the model. Because both of these efforts involve rather lengthy and detailed analysis only the highlights will be presented in this letter. The more detailed discussions are presented in the enclosure to this letter.

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The implementation of the new log likelihood ratio is based on a restatement of the problem as it pertains to the null hypothesis. This restatement involves consideration of the distribution of the maximum value of all independent samples of noise alone in the tracking window instead of the distribution of the amplitudes of these samples. The result of this is a slightly new form for the log likelihood ratio. Specifically, whereas the old log likelihood ratio employed a "degradation" factor* of log N_{AV}, where N_{AV} is the average number of noise-only preliminary threshold crossings, the new log likelihood ratio utilizes a factor of log N, where N is the number of independent samples of noise alone in the tracking window. Although the difference may appear small, the experimental results indicated a marked difference in computer loading and track intensity.

Figures 1 and 2 show the results of implementing the new log likelihood ratio as compared to the baseline system. It is evident from Fig. 1 that the number of status units and hence computer loading are significantly reduced by the new likelihood ratio. In particular, for a small number of pings -- indicating little time from initiation of the SLR algorithm -- the number of status units is reduced by approximately one-half. For intermediate to larger numbers of pings, the average number of status units is reduced by about one-third. It is also worth noting that the tendency for the average number of status units to increase is not present when the new log likelihood ratio is used. Figure 2 shows the average track intensity as a function of ping number. These results indicate that the average track intensity using the new log likelihood ratio is increased somewhat. This is probably a result of thresholding that is performed internal to the SLR algorithm

*This degradation factor is a quantity by which the log likelihood ratio is reduced on each echo cycle. Its purpose is to ensure that noise alone tracks will not build up and cause excessive clutter on the display.

rather than being a true gain in target track intensity relative to average clutter intensity and density. Therefore, this result should be interpreted as evidence of there being no loss in target detectability due to use of the new log likelihood ratio.

It may thus be concluded that the use of this modified likelihood ratio gives a significant reduction in computer loading with no attendant degradation in target track intensity.

The model used for computing marking probabilities treats a limited problem in which no new tracks of noise alone or signal plus noise are allowed to start after the first ping cycle. The results of this procedure make it possible to study the effects of changes in various SLR parameters cheaply and quickly, and with more accuracy than before. The theory upon which this model is based and the detailed results are given in the accompanying enclosure.

Thus far, the model has been used to investigate the detection performance of the SLR algorithm. The conditions assumed for this investigation are the following. First, the modified form of the log likelihood ratio is treated. Second, the size of the track window is taken to be large enough to encompass the maximum motion in both bearing and range of a modern, maneuvering submarine during an interping period. Thirdly, Doppler is assumed to be unavailable for decision and tracking purposes. The assumption of maximum target motion in range and bearing during an interping period results in rather large volumes of suspicion -- or tracking windows. This in turn means that, in general, there is a large number of noise alone samples which must be considered for SLR processing along with the signal-plus-noise sample. If, for the purpose of reduced computer loading, the algorithm is designed to accept as the track the largest log likelihood ratio from all of the resolution cells within the track window, it becomes an important matter to investigate the probability that the maximum of all the noise sample log likelihood ratios will exceed the log likelihood ratio of the signal-plus-noise sample. The reason that this question is important is that the track coordinates within a window are those of the sample whose log likelihood ratio is the largest. If this sample is a noise sample, the coordinates will be

incorrect. This means that the projected location of the track on the next echo cycle will be incorrect and hence that a spurious noise track will have been generated. This produces loss of target, increased clutter on the display and increased computer loading.

Now in order for all of this to occur, there must be an appreciable probability that the maximum of the noise sample log likelihood ratios will be greater than the signal-plus-noise log likelihood ratio. This probability is a function of both the track window size -- i.e., the number of range-bearing resolution cells within the track window, and the design signal-to-noise ratio. The analysis, which is presented in the enclosure, shows that for the large track windows, which one must use with high resolution sonars and fast targets, the probability of the maximum noise sample exceeding the signal-plus-noise sample becomes appreciable (> 0.5) when design signal-to-noise ratios of approximately ≤ 8 dB are used. With greater design signal-to-noise ratios [$(S/N)_D \approx 12$ dB] the situation is reversed in the sense that the probability of the maximum noise sample exceeding the signal-plus-noise sample becomes small, i.e., < 0.5. Thus, it is concluded that when the track windows are large, it does not appear feasible to attempt to design the SLR for optimally detecting targets with signal-to-noise ratios much lower than 12 dB. That is, the use of design signal-to-noise ratios less than 12 dB does not appear advisable under these conditions. It is possible, however, that targets with signal-to-noise ratios less than the design signal-to-noise ratio can be detected if sufficient time is allowed. In addition, it is quite likely that if additional information such as Doppler were used it would be reasonable to consider detection of targets with smaller signal-to-noise ratios.

In the next quarter, the study of further possible modification to the log likelihood ratio will be studied. Although this has been carried to some extent in the work reported above (by the

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inclusion of a new value in the log likelihood ratio), this additional study will seek an equation that may give better rejection of noise and faster acceptance of signal-plus-noise tracks. Also, the procedure used to calculate clutter probabilities will be refined in order to extend it to the more general SLR processor.

Very truly yours,

Hugh A. Reeder

Hugh A. Reeder
Project Director

HAR:ca

Enclosure

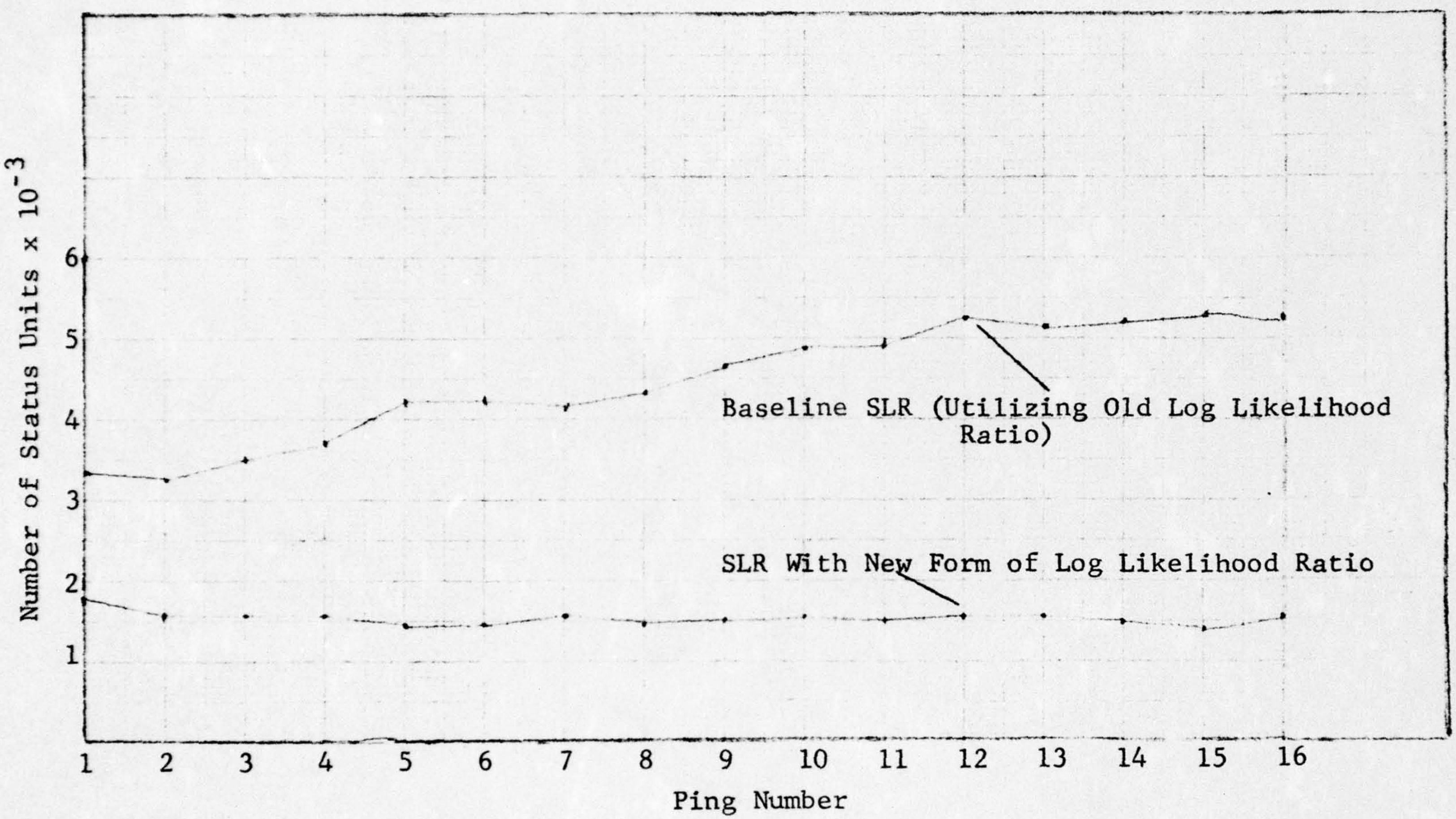

Fig. 1 NUMBER OF STATUS UNITS GENERATED VS PING NUMBER NOISE ONLY CONDITION, UNLIMITED LINKAGES, DESIGN S/N = 12 dB.

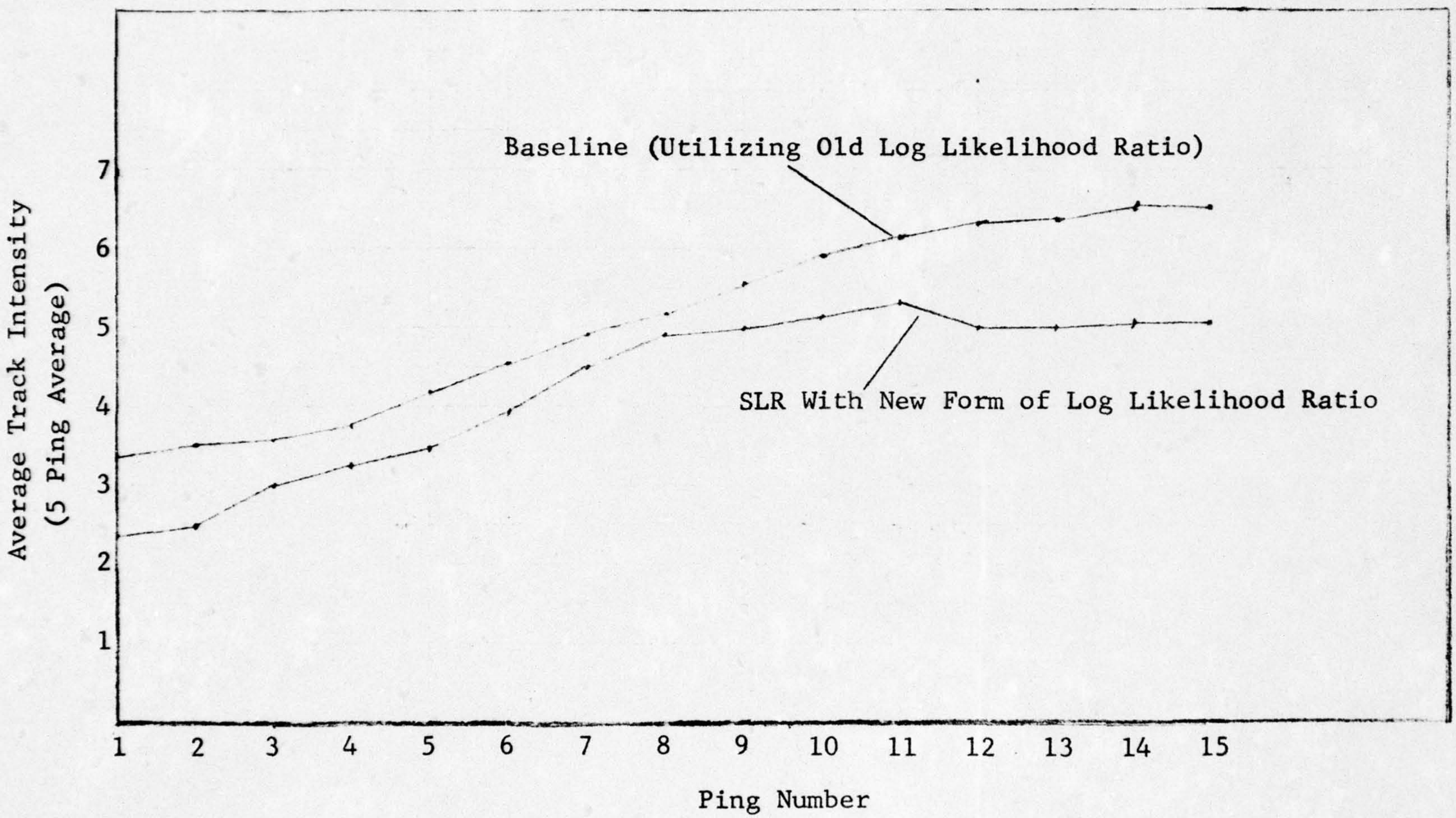

Fig. 2 AVERAGE TRACK INTENSITY VS PING NUMBER (NOISE ALONE CONDITIONS). CURVE 1 IS BASE LINE DATA. CURVE 2 IS DATA USING NEW LIKELIHOOD FACTOR (FAC = 1).

1. INTRODUCTION

When the SLR processor was first implemented using single beam data, tracks based on noise were quickly rejected and after a few pings both the number of tracks due to noise and the clutter rate were relatively constant. When the SLR processor was modified and applied to multibeam data with fairly narrow tracking windows (volumes of suspicion) the same results were observed although more pings were required to reach a steady state and both the number of noise tracks and the clutter rate for any given threshold were increased. In the baseline studies under the present contract the tracking windows of the multibeam case were increased considerably in order to simulate tracking a more realistically maneuvering target. When using these larger windows the number of noise tracks continued to increase with each ping cycle until about ping 13, after which the number of noise tracks was relatively steady with perhaps a slight increase through ping 16, the last observed. The clutter rate observed seemed to exhibit about the same trend, that is, increasing through ping 13 with only a slight increase thereafter. The following explanation is offered for this phenomenon.

When these large tracking windows are used, the probability that a noise sample of significant amplitude will occur in the window becomes so great that it is almost certain that once a track is started it will continue if the design signal-to-noise ratio is low enough (in this case 12 dB). Because noise tracks are not rejected, the number of noise tracks increases. Similarly, the clutter rate increases because the log likelihood ratios associated with the noise tracks tend to integrate up. Since, in this study, display thresholds are set to maintain constant clutter density and intensity, this increase in number and magnitude of noise tracks requires that the display thresholds be increased in order to maintain constant clutter. The increased display thresholds mean that the probability that the log likelihood ratio associated with

a signal-plus-noise track will cross a given display threshold is decreased relative to the probability of this event occurring had the track windows been smaller. This, of course, causes a decrease in track intensity.

In addition to this effect of increased track window size, there is another mechanism which aggravates the situation. This is the use of an upper limit placed on the value of the log likelihood ratio. This limit was established to prevent spurious tracks of significant size from branching off tracks with very large log likelihood ratios. That is, if the log likelihood ratio were allowed to reach a high enough value, every sample it linked to would be treated as a significant track. Also, if the target returns suddenly faded, a track with an excessive log likelihood ratio would be retained by the SLR processor for many more pings than it should be (rejection of intermittent tracks is based on degredation of the log likelihood ratio). This upper limit on the value of the log likelihood ratio tends to cause both the number of noise tracks and the clutter rate to reach an approximate steady-state condition. Since the ability of the signal-plus-noise track to integrate up has been interferred with and the noise level has risen (because of larger track windows), signal detection is decreased.

Reduction of computer loading was, at first, considered the highest priority and previous investigation treated methods which significantly reduced computer loading by limiting the number of linkages. Linkages with the largest log likelihood ratios were retained because they were most likely to be the true target tracks, and by retaining the maximum noise tracks, the clutter problem was retained. Hence, a new approach had to be devised to cope with the problem of the rise in clutter rate.

2. BASIS FOR THE PROBLEM

In order to obtain a better idea concerning this phenomenon the probability that the maximum noise sample in a tracking window exceeds a sample from a signal-plus-noise track of a given S/N was calculated and plotted in Fig. 1 for various window sizes. The curve marked N = 1 corresponds to a single sample of Rayleigh noise at the output of a linear correlator. The curve marked N = 243* corresponds to the large volume of suspicion, i.e., when no range rate information is available, that was used in the display comparison in the Summary Report[2] and the baseline data of the present contract. This volume of suspicion can handle targets that have range rates up to ±20 knots and bearing rates corresponding to one beam per ping period. In this case, this means the large volume of suspicion has 243 resolution cells, each cell containing one independent sample. The curve marked N = 51 corresponds to the small volume of suspicion, i.e., when some range rate history is available. This volume is designed to track targets which have a rate of change of range rate of less than ±4 knots per ping cycle and move one beam per ping cycle in bearing. This volume contains 51 independent samples. The curves marked N = 41 and N = 5 correspond to the single beam case reported in the first SLR report[3].

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*These values of N derive from the assumption of typical values of sonar parameters and target dynamics. For example, the time resolution, ping repetition period, and range scale were chosen from a typical operational sonar while the target dynamics were based on the maneuverability of modern submarines.

[2]Reeder, Hugh A. and H. D. Record, "Computer-Aided Detection (U)," Summary Report, TRACOR Document 68-1238-C, 19 November 1968.

[3]Reeder, H. A., "Computer Utilization of Sequential Hypothesis Testing for the Detection and Classification of Sonar Signals (U)," TRACOR Document 67-717-U, 7 December 1967.

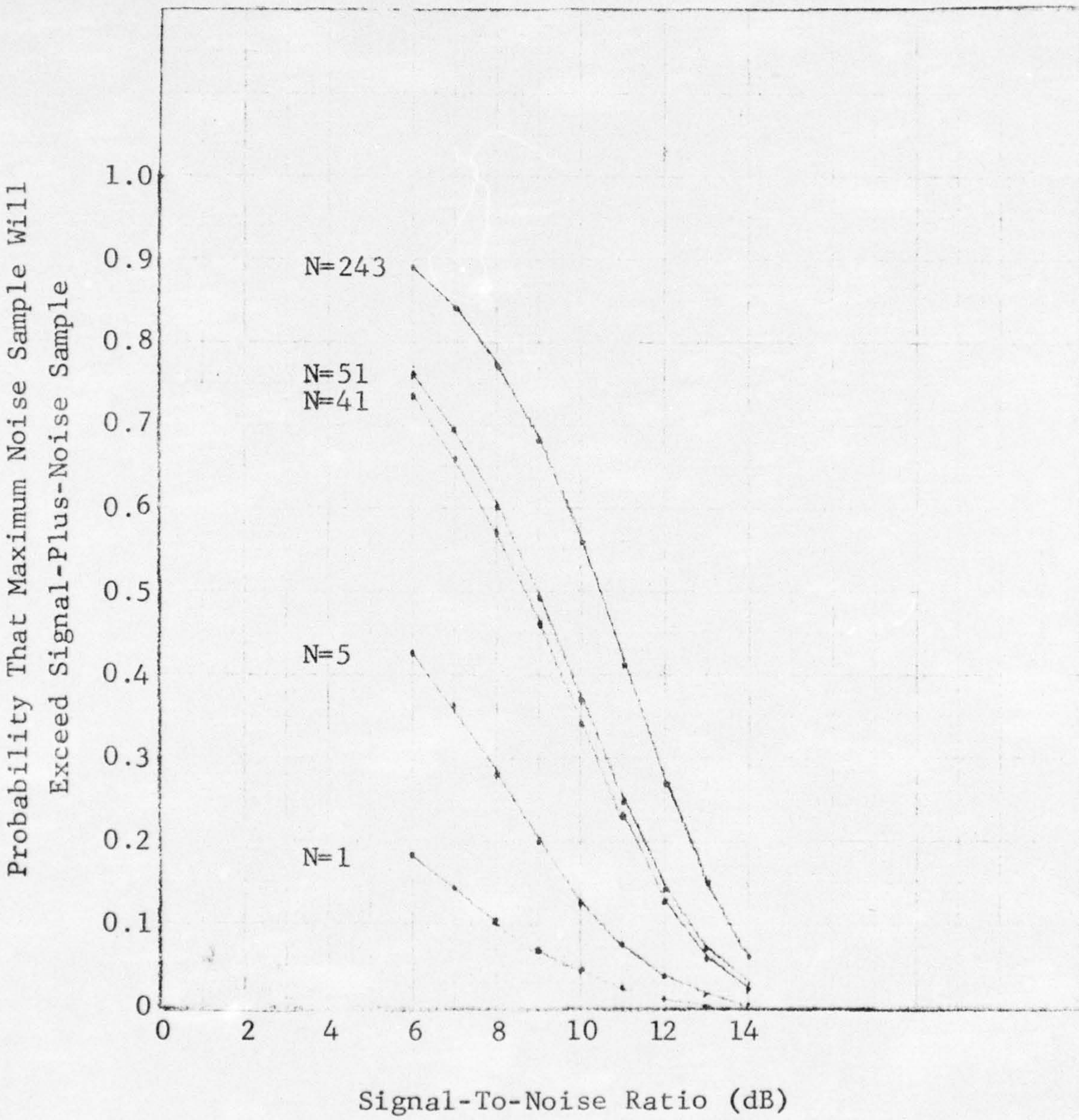

Fig. 1 PROBABILITY OF MAXIMUM NOISE SAMPLE EXCEEDING SIGNAL-PLUS-NOISE SAMPLE FOR VARIOUS SIZES OF WINDOWS. (N EQUALS NUMBER OF INDEPENDENT SAMPLES IN THE WINDOW)

Returning to the baseline parameters of N = 243 and N = 51, Fig. 1 shows that the maximum noise sample in the large volume of suspicion will exceed the 12 dB signal- plus-noise sample with probability 0.27 and for the small volume with probability 0.14. These relatively low probabilities mean that the noise tracks can be separated from the signal-plus-noise tracks fairly quickly based on amplitude alone, i.e., exclusive of other information such as Doppler. However, with an 8 dB signal the situation is not the same. The probability that the maximum noise sample in the large volume of suspicion is greater than a signal-plus-noise sample is about 0.77. In the small volume this probability is 0.60. Therefore, in either case it is more probable that the maximum noise sample in the tracking window will be larger than the sample from the actual target track. Since the maximum noise sample is usually larger than the sample from an 8 dB target track and the decision test of the SLR processor is based on a threshold crossing by a linear function of the amplitudes of the observed track samples, the SLR processor will usually decide that the noise track based on linking the maximum noise sample in the window is signal plus noise if the test is designed to accept 8 dB targets. That is, if a group of samples taken from an 8 dB track can be expected to yield a statistic to cross a decision threshold, a group of samples which are usually bigger than the track sample can be expected to cross the same threshold. This acceptance of noise tracks as signal plus noise causes a high clutter rate. Alternately, if the SLR processor is designed to reject the noise track based on the maximum noise sample in the window, the 8 dB target track will more than likely be rejected because its samples are usually smaller than the maximum noise sample track.

Perhaps, another approach will clarify these ideas. If the random variables and the statistic used in the SLR processor are restricted to a finite domain and range, and the number of track samples, N_{max}, limited to some finite number (physically, this is the case), a basic inequality may be used[4].

$$\frac{E(u(x)) - T}{M} \le P(u(x) \ge T) \le \frac{E(u(x))}{T} \tag{1}$$

where x denotes the track samples $x_1, x_2, \ldots, x_n$ for n pings,
u(x) is the SLR statistic, the log likelihood ratio,
E(u(x)) is the expected (average) value of u(x),
T is a decision threshold on u(x), and
M is the largest value that u(x) is allowed to take*.

In the case at hand, u(x) takes on the particularly simple form of a sum of independent random variables. The expected value is given by

$$E(u(x)) = E(a \sum_{i=1}^{n} x_i + c(n))$$

$$= a \sum_{i=1}^{n} E(x_i) + c(n)$$

where "a" is a constant (determined by the design signal-to-noise ratio), and
c(n) is a function that depends on n, the number of pings that the track has continued, and the design signal-to-noise ratio.

[4] Loeve, M., Probability Theory, Van Nostrand, Princeton, New Jersey, Third Edition, P 157.

*The notation above differs somewhat from the reference. The fact that a finite domain and range is considered and that u(x) is a continuous function insure that the integrals necessary for the proof exist

For the baseline parameters the expected value of $u(x)_N$ the track statistic based on the maximum noise sample in the window, is

$$
\begin{aligned}
E(u(x)_N) &= 1.25\ a + c(1) && n = 1 \\
&= 4.72\ a + c(2) && n = 2 \\
&= (4.72 + 2.98\ (n-2))\, a + c(n) && n = 3,4,\ldots,N_{max}
\end{aligned}
$$

and the expected value of $u(x)_S$, the track statistic for an 8 dB target, is

$$E(u(x)_S) = 2.72\ a + c(n) \qquad n = 1,\ 2,\ \ldots,\ N_{max}$$

By comparing the above it is found that for $n \geq 5$

$$E(u(x)_N) \geq E(u(x)_S). \tag{2}$$

The left hand side of inequality (1) may be used to choose a threshold, T, such that, after a given number of pings, the probability that the SLR statistic for signal plus noise will be greater than T is a set amount, for example, 0.5. If the number of pings is greater than or equal to 5, inequalities (2) and (1) imply that the probability that the statistic based on the maximum noise sample track will cross the chosen threshold has a greater lower bound than the signal case does. This tends to indicate that if this threshold is used a high false alarm probability can be expected. If the right hand side of inequality (1) is used, a bound may be set on the false alarm probability of the track based on the maximum noise sample in the windows. Inequality (2) implies that the probability that the statistic in the signal case has a lower upper bound than the noise case for any threshold T. This tends to decrease or limit signal detection.

Working with an inequality such as (1) is not very satisfying. One would like to state sharper results such as: the probability that the statistic of a noise track based on the maximum noise sample in the window crosses a given threshold is greater than the probability that the statistic of a signal-plus-noise track will cross the same threshold. This is certainly more explicit than a statement about lower bounds which tends to indicate the desired implication but does not prove it. Unfortunately, to show the desired results one must evaluate some very complicated integrals. A study that is related to this problem and may eventually offer a solution is reported in section 4 of this enclosure.

By narrowing the tracking windows, the probability that the maximum noise sample in the window will cross a given threshold is decreased and a lower signal-to-noise ratio track may be detected without increasing clutter rate excessively. By including additional data, such as Doppler, the noise tracks may be eliminated on that basis, also, thereby decreasing computer loading and clutter. Hence, lower signal-to-noise ratio targets may be detected. It should therefore be noted that the implication that the SLR processor cannot track signals below 8 dB is dependent on the window size and the fact that only amplitude information is used.

3. A SOLUTION OF THE PROBLEM

Since a major problem in computer loading and display clutter is that the track based on the maximum noise samples in the volumes of suspicion is being accepted as signal plus noise, it appears that the sequential test should be modified. A previous method[5] used for controlling loading was to subtract the logarithm of the average number of samples above the preliminary threshold in the tracking window. This was justified by assuming there would be at most only one true target track in each tracking window. Then, the probability that the link under consideration was in fact the true track, given there is a target present, was 1 divided by the average number, N_{av}, of links (number of preliminary threshold crossings) considered. The probability that the link was noise under the null hypothesis, i.e., noise alone is present, is 1. Hence, the likelihood ratio $\ell'(x)$ is $\ell'(x) = 1/N_{av}/1 = 1/N_{av}$ and the log likelihood ratio $L'(x) = -\log N_{av}$. N SUB AV.

The proposed new likelihood ratio would again make use of the assumption that only one true signal-plus-noise track could be present. The null hypothesis is replaced with the hypothesis that the sample is the maximum sample in the tracking window due to noise alone. The probability density function, $p_m(x)$, for this maximum sample is given by

$$p_m(x) = N[P(X<x)]^{N-1}\, p_n(x),$$

where $p_n(x)$ is the noise density function, $P(X<x)$ is the noise cumulative distribution function and N is the number of independent samples in the tracking window.

[5]Reeder and Record, op. cit., p. A-7.

The density function, $p_{s+n}(x)$, for the signal-plus-noise hypothesis is unchanged. The new likelihood ratio, $\ell'(x)$, is given by

$$\ell'(x) = \frac{p_{s+n}(x)}{N[P(X<x)]^{N-1}\ p_n(x)}$$

$$= \frac{1}{N[P(X<x)]^{N-1}}\ \ell(x) \ ,$$

where $\ell(x)$ is the previous likelihood ratio based on a single sample. Taking logarithms,

$$L'(x) = L(x) - \log N - (N-1) \log P(X<x) .$$

Now as x increases $P(X<x)$ approaches unity and $\log P(X<x)$ approaches 0; hence, for large X the last term may be neglected.

In order to examine this, several graphs were run out plotting $L'(x)$, $L(x)$ and the approximation to $L(x)$ for Rayleigh-Rice statistics. In Figs. 2 through 6, the log likelihood is plotted against the amplitude in units of noise standard deviation above the noise mean. The volumes of suspicion are the same as previously described and the design signal-to-noise ratios are 6, 8, 10, 12, and 14 dB, respectively. On each graph, for large amplitudes, $L'(x)$ for both large and small windows becomes parallel to $L(x)$, the solid line. On each graph the $L'(x)$ are shifted down by log N, where N is the number of independent samples in the window. Hence, for large amplitudes the approximation $L'(x) = L(x) - \log N$ is adequate. However, at low amplitudes an interesting thing occurs. The log likelihood ratio increases again. This means that the signal-plus-noise density function is larger than the maximum noise sample density function.

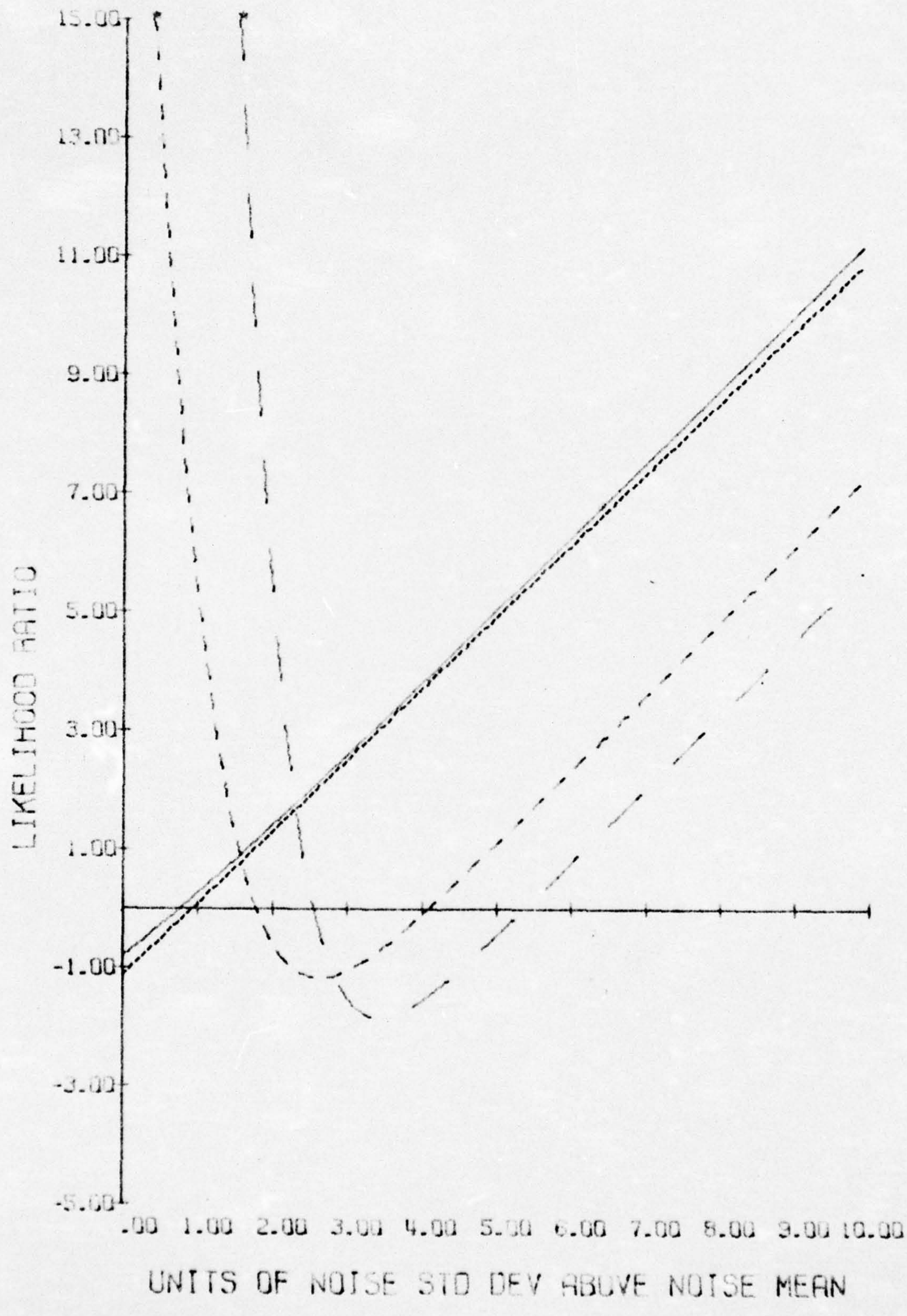

Fig. 2 LIKELIHOOD RATIO FOR RAYLEIGH NOISE(SOLID). MAX SAMPLE IN SMALL WINDOW(SHORT DASH). AND LARGE WINDOW (LONG DASH). APPROXIMATION(DOTS). DESIGN S/N= 6DB

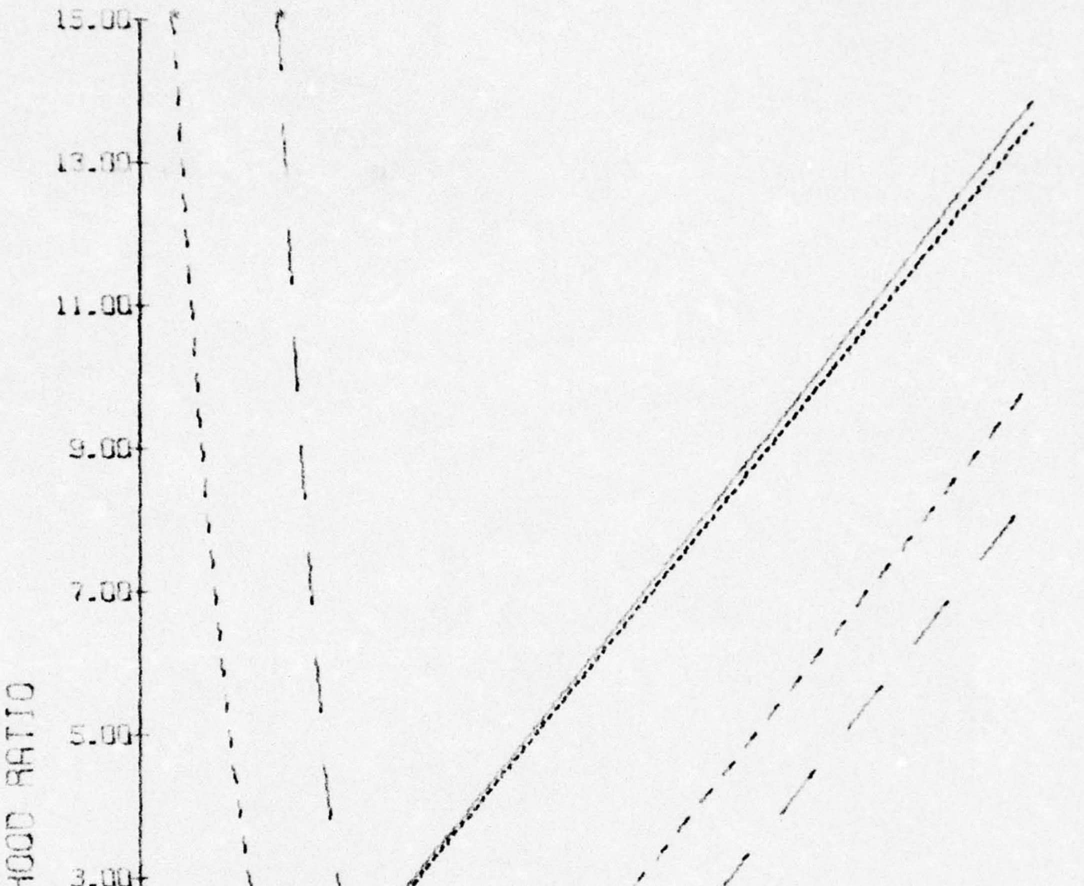

Fig. 3 LIKELIHOOD RATIO FOR RAYLEIGH NOISE(SOLID). MAX SAMPLE IN SMALL WINDOW(SHORT DASH). AND LARGE WINDOW (LONG DASH). APPROXIMATION(DOTS). DESIGN S/N= 5DB

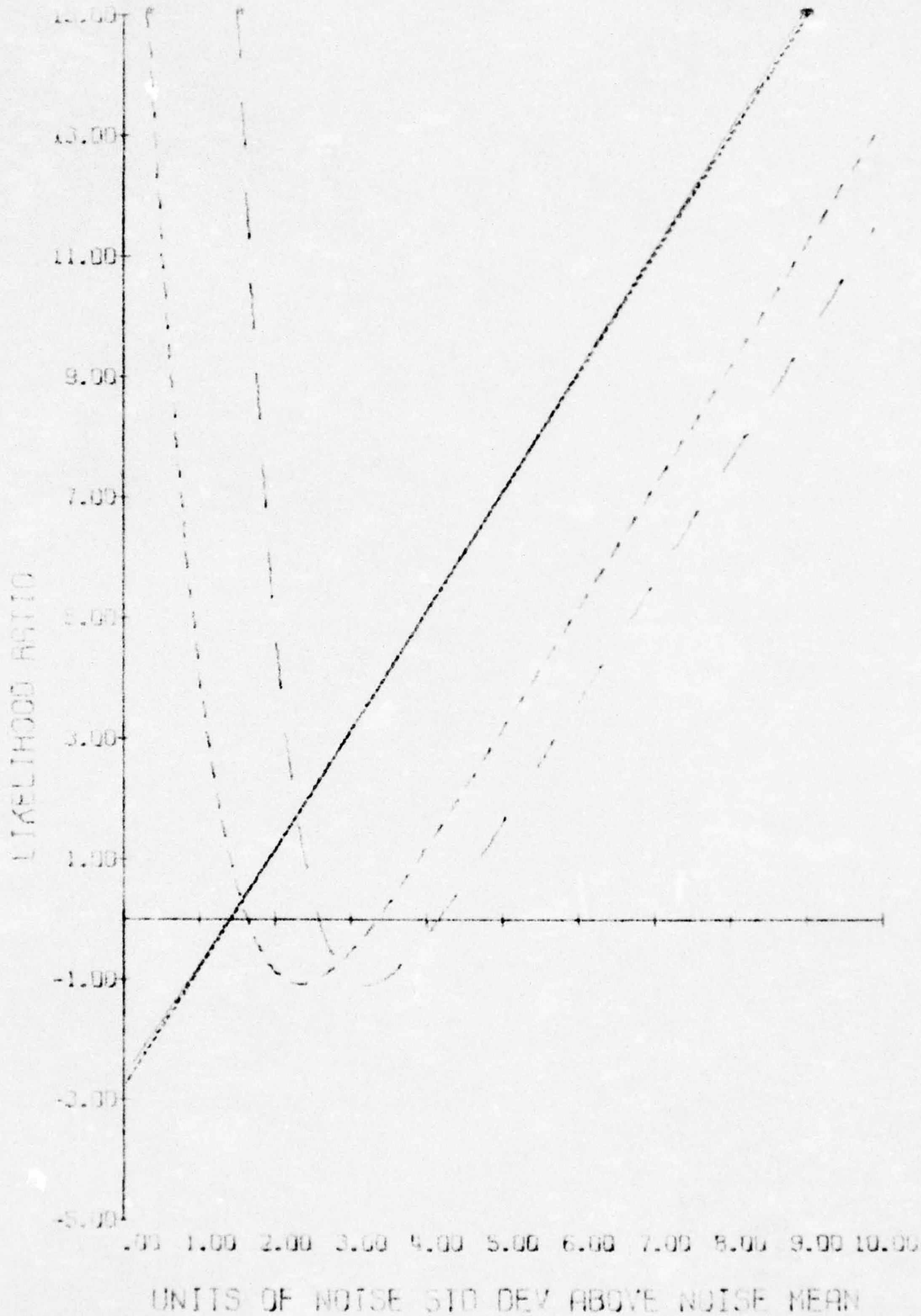

Fig. 4 LIKELIHOOD RATIO FOR RAYLEIGH NOISE(SOLID). MAX SAMPLE IN SMALL WINDOW(SHORT DASH), AND LARGE WINDOW (LONG DASH). APPROXIMATION(DOTS). DESIGN S/N= 10DB

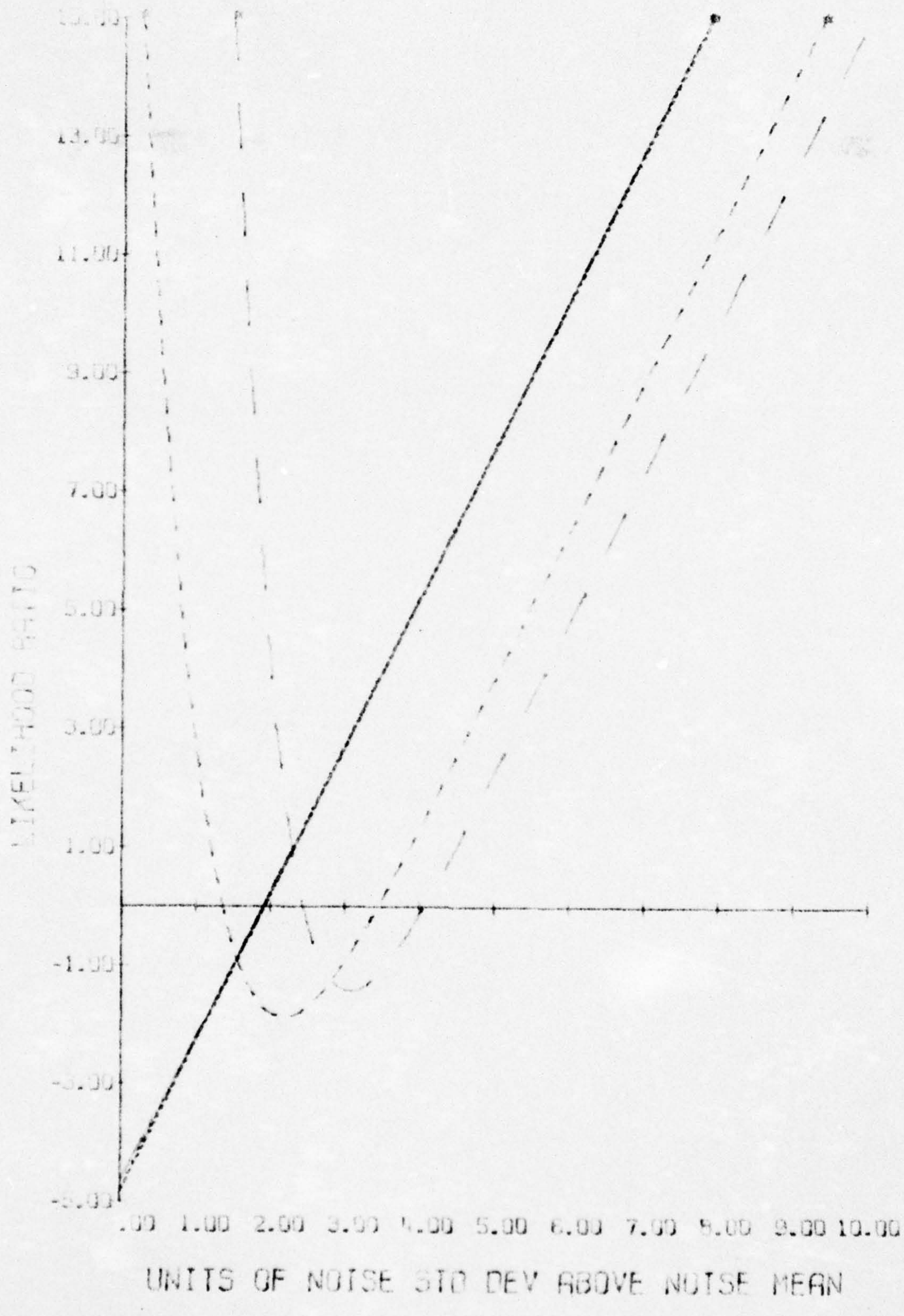

Fig. 5 LIKELIHOOD RATIO FOR RAYLEIGH NOISE(SOLID). MAX SAMPLE IN SMALL WINDOW(SHORT DASH), AND LARGE WINDOW (LONG DASH). APPROXIMATION(DOTS). DESIGN S/N= 10DB

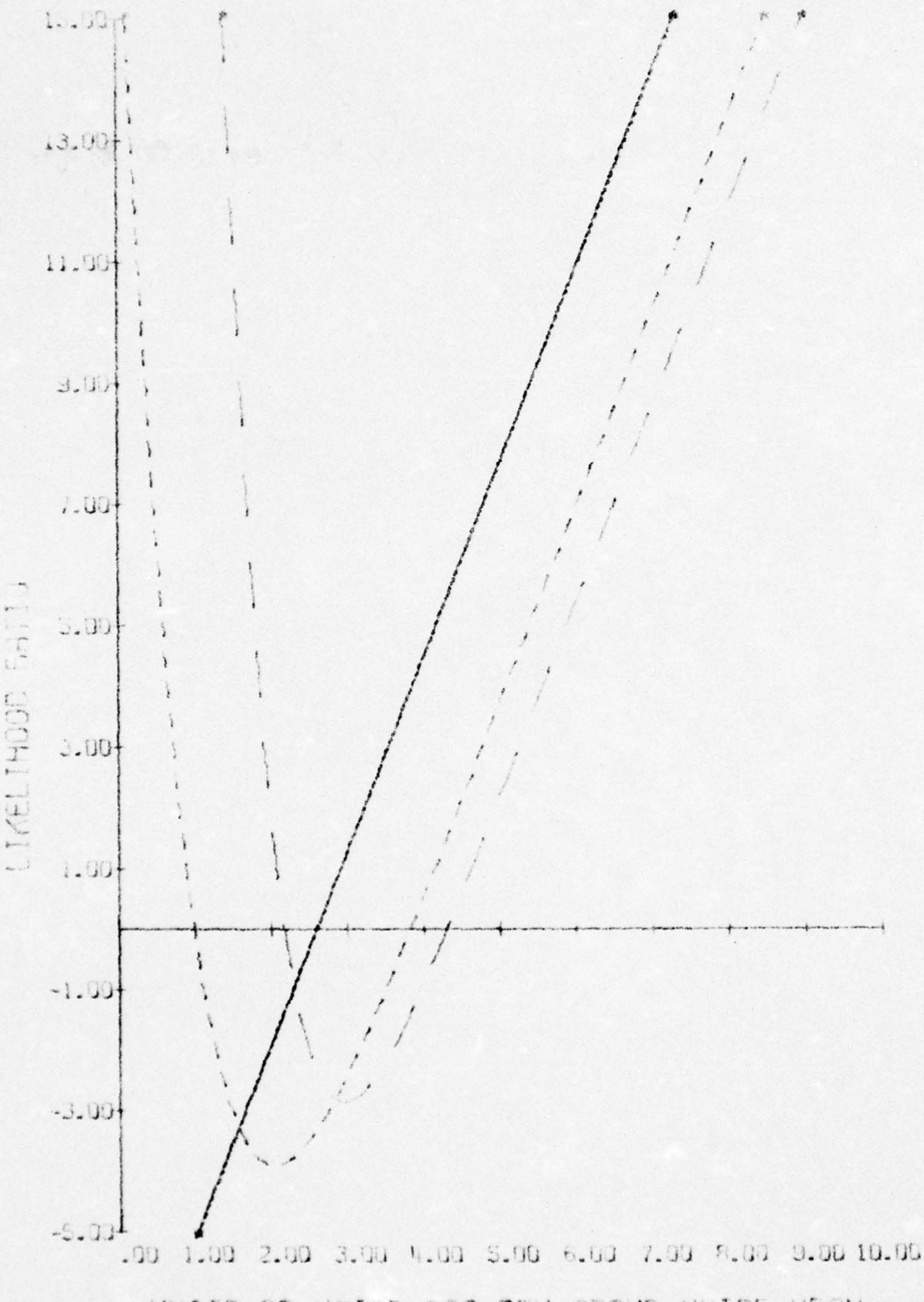

Fig. 6 LIKELIHOOD RATIO FOR RAYLEIGH NOISE(SOLID). MAX SAMPLE IN SMALL WINDOW(SHORT DASH). AND LARGE WINDOW (LONG DASH). APPROXIMATION(DOTS). DESIGN S/N= 14DB

Since the SLR processor will allow linkages with other than the maximum sample in the window, it would be incorrect (and disastrous in terms of clutter rate and computer loading) to use the true likelihood ratio for all linkages. Therefore, the simplest and most practical solution is to use the present linear approximation to the log likelihood ratio and subtract off log N instead of log N_{av} for all amplitudes. This approximation will be accurate for sonar processor output samples greater than 2.5 to 3 noise standard deviations above the noise mean. For a design signal-to-noise ratio of 12 dB, the probability that the maximum noise samples in the window will be in this region is about 0.5. A 12 dB signal will be in the region 89% of the time and a 9 dB signal 54% of the time.

4. RESULTS OF USING THE NEW LIKELIHOOD RATIO

The true merit of using this new factor in the log likelihood ratio may be seen by comparing the baseline data based on subtracting log N_{av} with the new data generated by subtracting log N. The results are presented in Figs. 7 and 8. (In all of these curves, the factor FAC = 0.21 indicates that log N_{av} was used and FAC = 1.0 means that log N was used*.) Figure 7 shows the number of status units generated for each of 16 consecutive ping cycles. The top curve is for the baseline processor and the lower curve is for the new L(X). Note that not only were the number of status units reduced significantly, but there was no general trend in the number of status units as exhibited by the baseline data. To obtain Fig. 8 the output of the new SLR processor was analyzed to obtain the distribution function, and thresholds found such that the probability of noise marking any given display level was the same for the baseline and new SLR processor. The lower curve in Fig. 8 represents the five ping running average of the baseline target track (target signal-to-noise ratio 12 dB, design signal-to noise ratio 12 dB). The top curve presents the results of new SLR processor run on the same data with the same design signal-to-noise ratio. The obvious conclusion is that the new SLR processor is superior in performance for these parameters since it gives a higher track intensity. The reason for this apparent gain in signal intensity is the decrease in noise clutter, allowing lower display thresholds which the signal-plus-noise samples cross with

*FAC is parameter of the SLR computer program. Its value was originally the probability that a single sample of noise would cross the initial input threshold of the SLR processor (FAC=0.21 in this case.) Hence, the average number of links in a tracking window, N_{av}, is given by FAC·N. Since log (FAC·N) was already used in the program, it was very easy to implement the new log likelihood ratio by simply changing value of FAC to 1.0, yielding log N instead of log N_{av}.

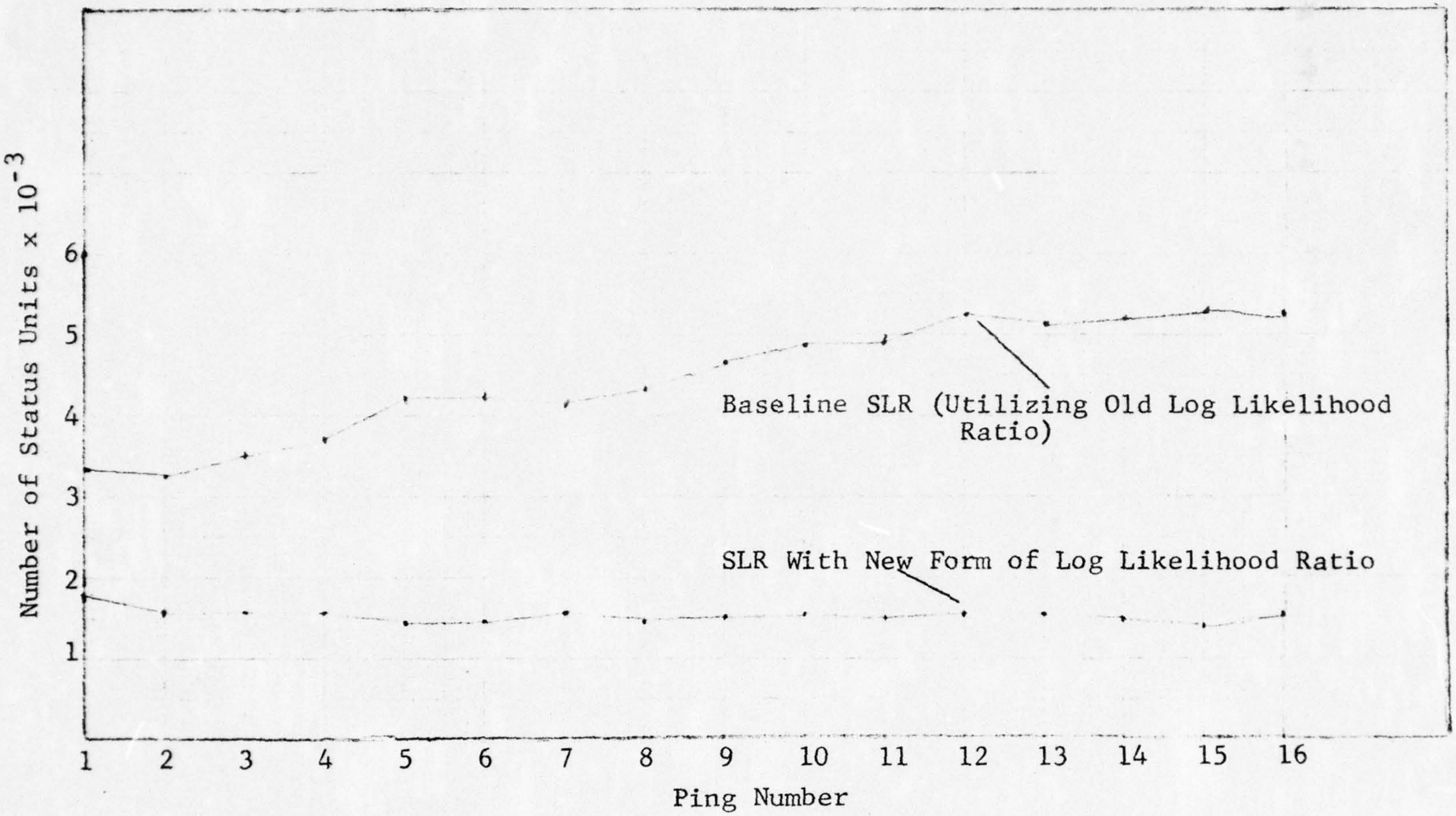

Fig. 7 NUMBER OF STATUS UNITS GENERATED VS PING NUMBER NOISE ONLY CONDITION, UNLIMITED LINKAGES, DESIGN S/N = 12 dB.

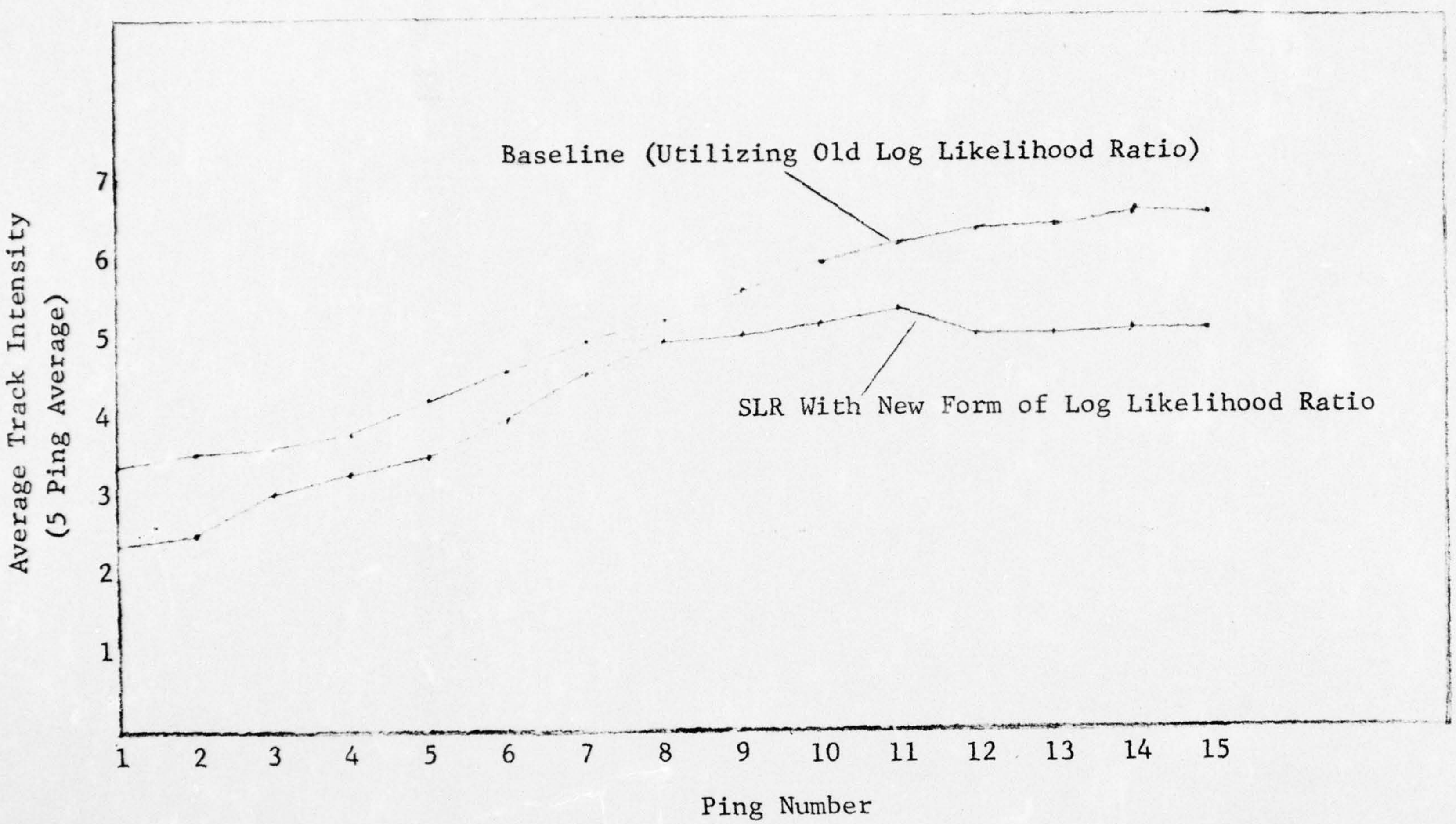

Fig. 8 AVERAGE TRACK INTENSITY VS PING NUMBER (NOISE ALONE CONDITIONS). CURVE 1 IS BASE LINE DATA. CURVE 2 IS DATA USING NEW LIKELIHOOD FACTOR (FAC = 1).

a higher probability. In summary, Fig. 7 and 8 show that using log N instead of log N_{av} gives a significant reduction in both computer loading and clutter rate as a function of ping number. This results in better detection capability on the display through an increase in track intensity.

If a higher track signal-to-noise ratio were used, the probability that the log likelihood ratio would cross a high display threshold becomes so near 1.0 in either case that differences would be masked. If very low track signal-to-noise ratios are used, the probability that the track samples are above the initial threshold is low enough that the track becomes intermittent and is difficult to enhance. There is a region just below 12 dB where tracks are reasonably continuous. For these signal-to-noise ratios one would expect the results to follow the 12 dB S/N results, but with less track intensity. Unfortunately, funds are not sufficient to investigate this region in the direct way that the 12 dB case was. Section 6 of this enclosure gives the results of a study which indicates to some extent the trend that the data can be expected to take in this region.

5. CALCULATION OF CLUTTER PROBABILITY

An analytical tool has been developed to study the SLR processor and to calculate clutter probabilities as a function of ping cycle. This tool is a computer program that calculates the probabilities of threshold crossing as a function of threshold. It can easily calculate probabilities of 10^{-6} or less, in a few seconds. To obtain accurate thresholds for such values would take many hours of computer time. This tool will be useful in determining the effects that changes in various parameters will have on track intensities and computer loading. The study of some of these parametric effects would not have been possible before because of the prohibitive computer costs.

The method is simple in concept. Let $p_i(L_1)$ be the probability that a single sample of L(x) is between $i\Delta L$ and $(i + 1)\Delta L$. Then the probability that the likelihood ratio, L_{12}, after the second ping cycle is between $i\Delta L$ and $(i + 1)\Delta L$ is

$$P_i(L_{12}) = \sum_{j=1}^{M} P_{i-j-k} P_j$$

where

M - corresponds to the maximum quantization level, and

k - is a constant based on log N.

Essentially the density of L_{12} is found by shifting the density of L_1 by log N and then convolving the results with the density of L_2. This density function for L_{12} may be summed to obtain the cumulative distribution function and the density function may be used to obtain the density function for the log likelihood after three pings and so forth. So long as ΔL is small the errors introduced in quantizing the probability density function will be small. The program is set up at present such that no new tracks are started after the first ping cycle. While this is a limitation, the results still provide useful information and work is continuing to extend the tool to include tracks that are continually starting.

The graphs for noise alone are presented in Figs. 9 through 16. Each graph has up to 10 numbered curves. Each curve represents the probability of exceeding threshold versus threshold after the indicated number of pings. New tracks are started on the first ping cycle only and the resulting tracks continued until the associated log likelihood ratio falls below the lower decision threshold. At the bottom of each figure the design signal-to-noise ratio is given along with the value of the parameter FAC. If FAC is equal to 1.00 the new log likelihood ratio utilizing log N was used. If FAC is .21, log N_{av} was used.

Figure 9 shows the results for the design S/N of 12 dB, utilizing the new log likelihood ratio. Note that the probability of a noise mark crossing a given threshold falls off with increasing ping number. This of course is a desired property since it means that noise tracks should be rejected rather quickly. Figure 10 shows the results under the same conditions as before except log N_{av} is subtracted off each time. In this case the probability that a noise mark will exceed a threshold increases with increasing ping number. This shows why there is a general increasing trend in the number of status units and the clutter rate in the baseline SLR processor. If additional pings are considered, the clutter tends to stabilize and is virtually unchanged after 16 pings. Figure 11 shows the same situation, but for a single beam set up similar to that used in the investigations reported in the first SLR report[6]. Since the windows are rather narrow, the probability that a significant noise sample is less and the unstable condition of rising clutter is not exhibited. It is only when the wide windows necessary for multiple beam operation in a slow pulse rate sonar* are used that this problem becomes important.

[6]Reeder, op. cit.

*Slow pulse rates give the target more time to move between track samples; hence, wider windows must be used.

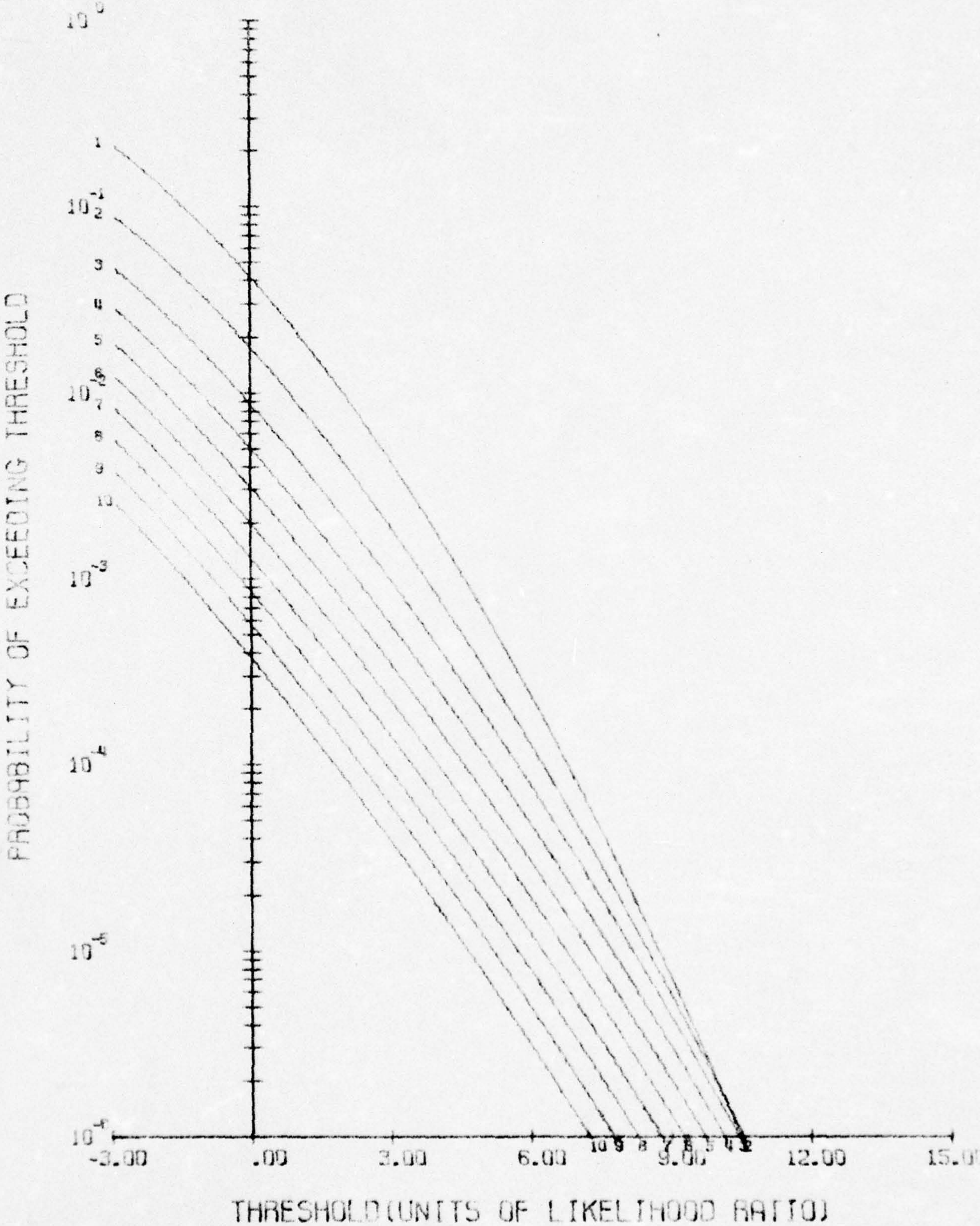

Fig. 9 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR NOISE ONLY. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=12 DB. FAC=1.00.

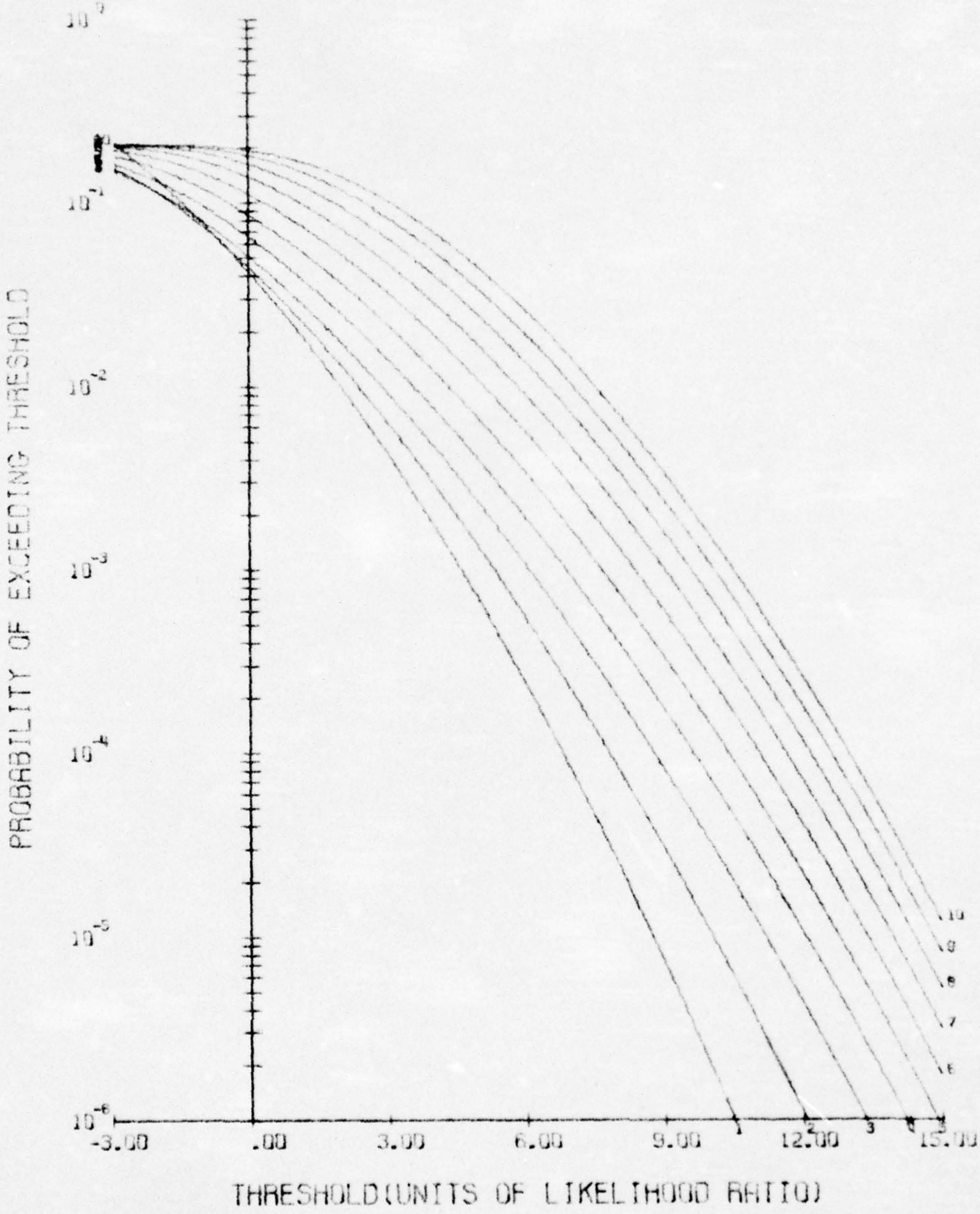

Fig. 10 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR NOISE ONLY. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=12 DB. FAC= .21.

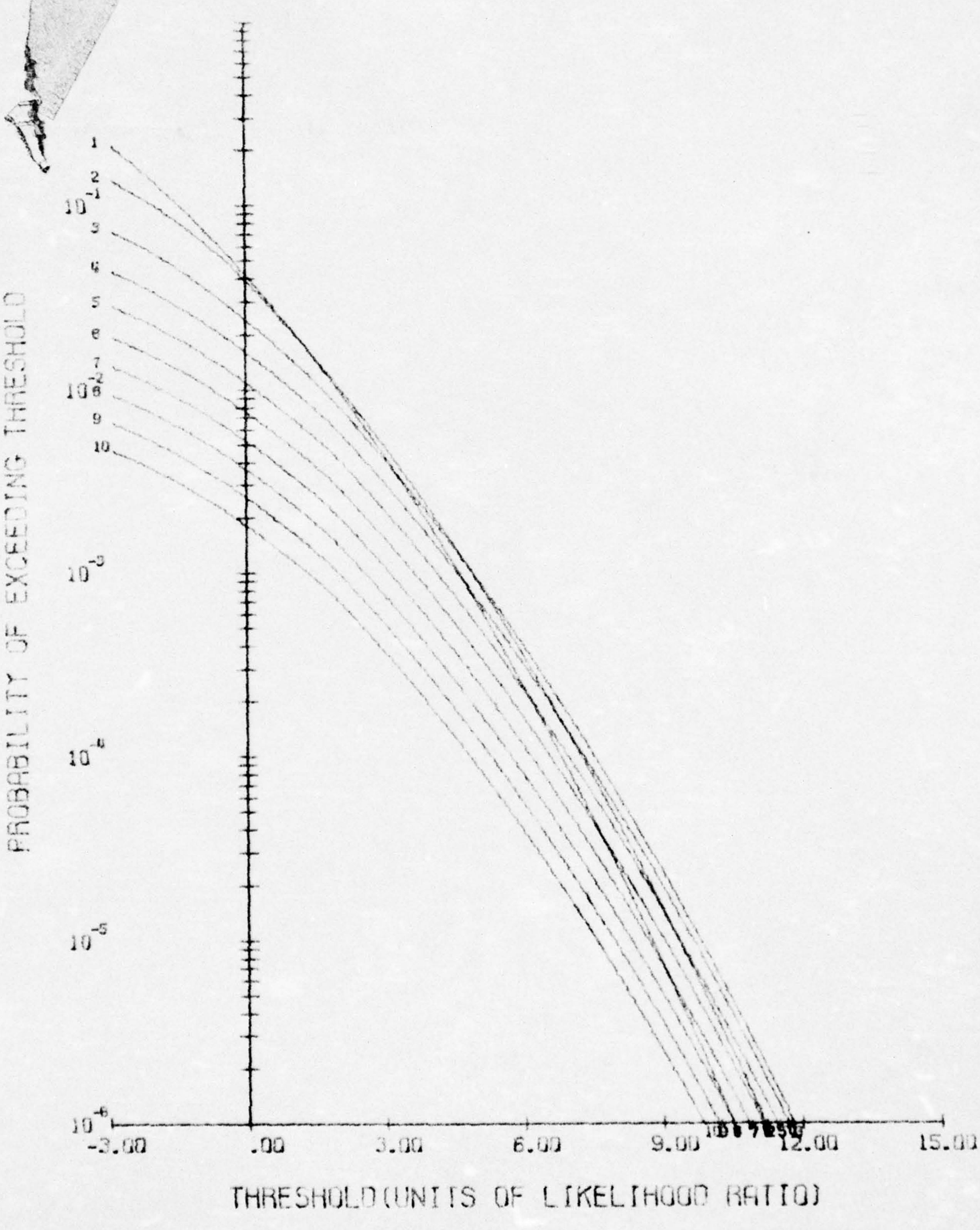

Fig. 11 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR NOISE ONLY. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=12 DB. FAC= .21.

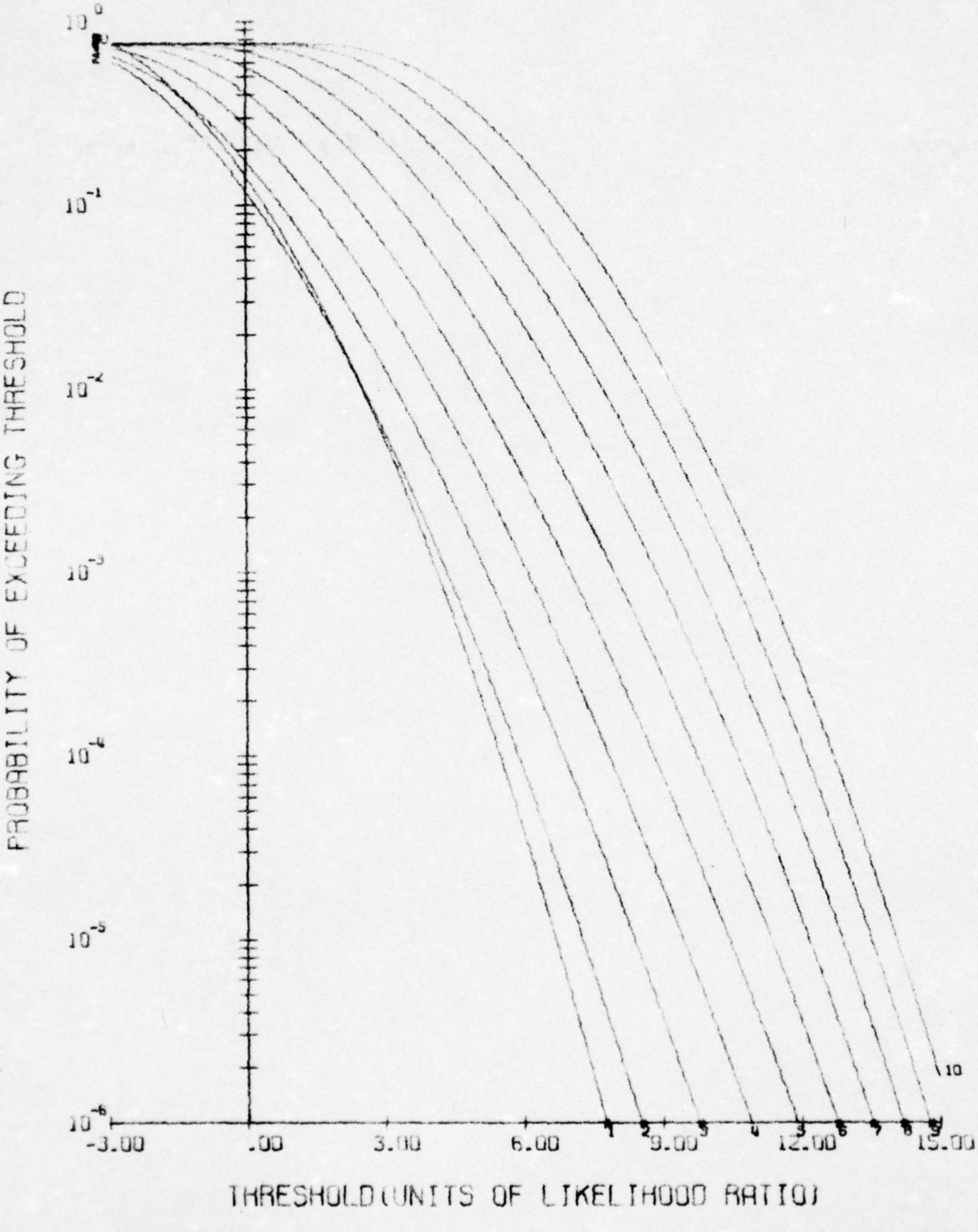

Fig. 12 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR NOISE ONLY. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N= 8 DB. FAC= .21.

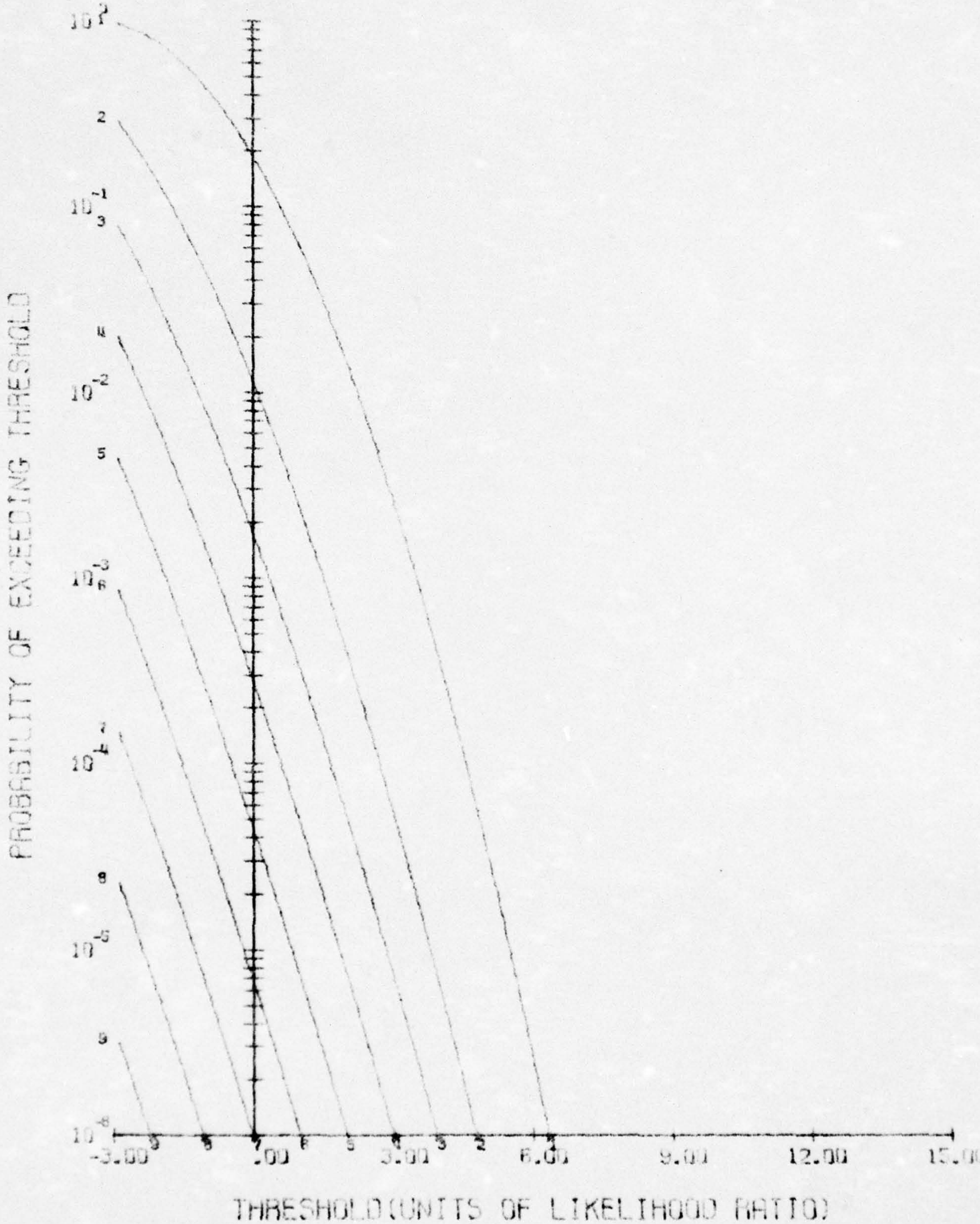

Fig. 13 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR NOISE ONLY. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N= 6 DB. FAC=1.00.

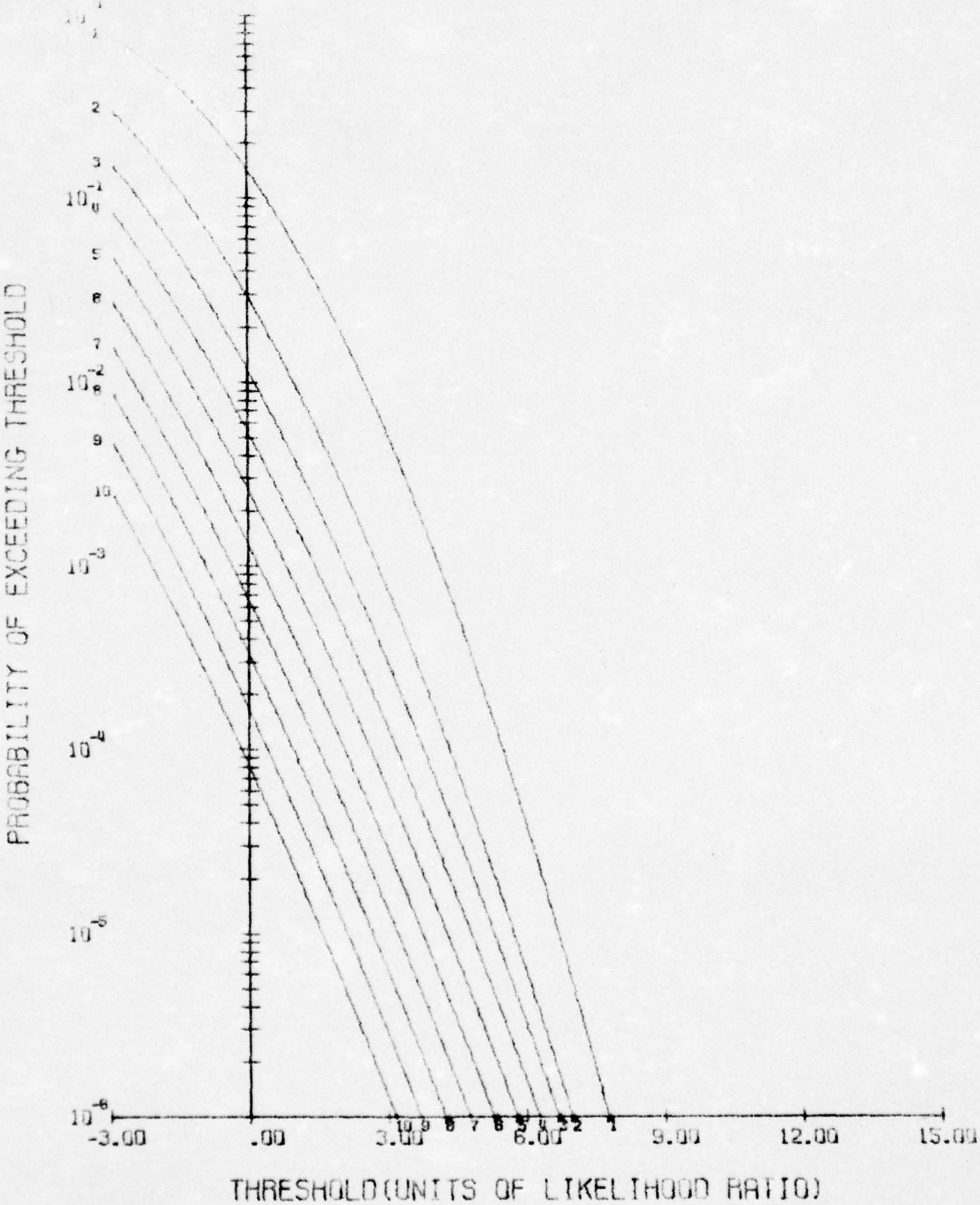

Fig. 14 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR NOISE ONLY. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N= 8 DB. FAC=1.00.

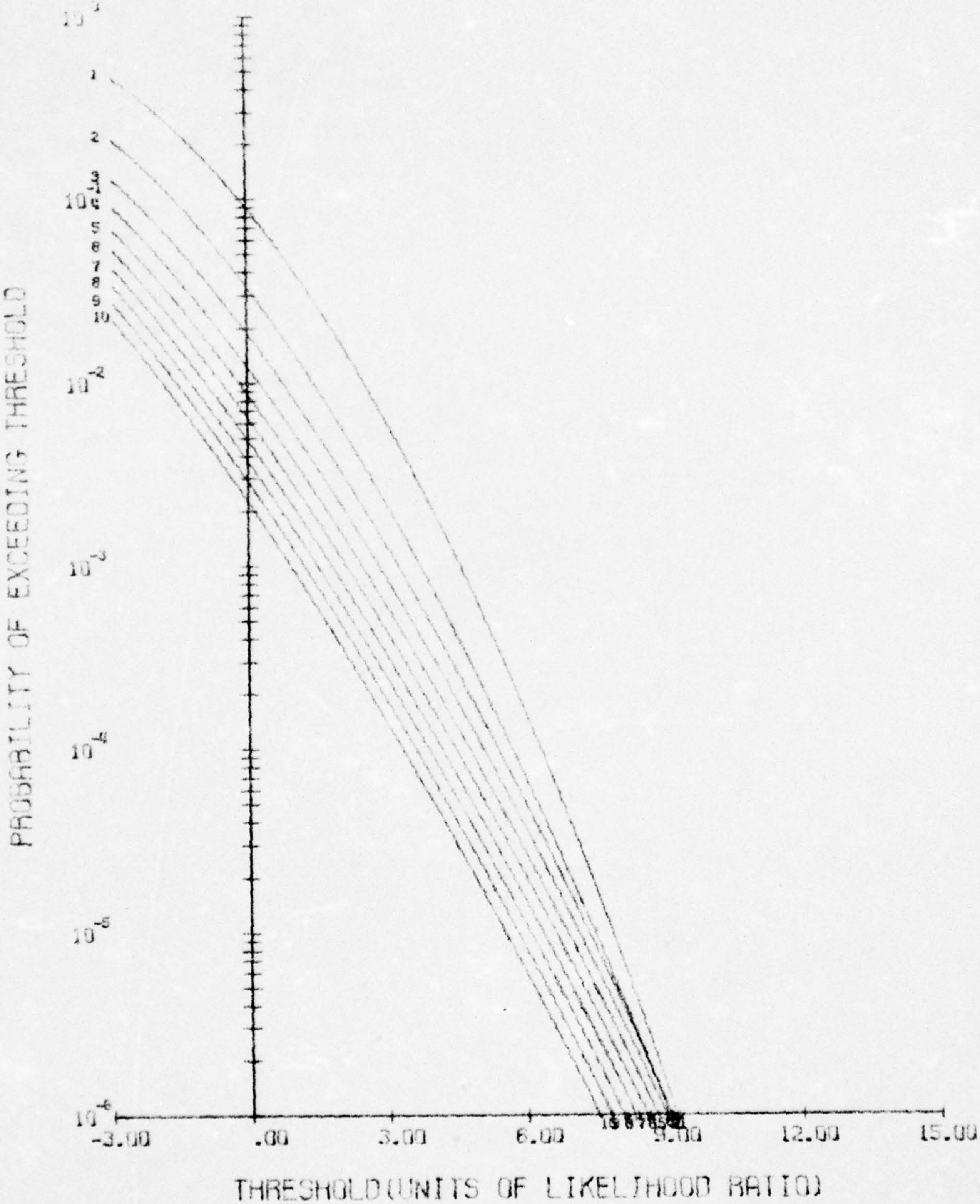

Fig. 15 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR NOISE ONLY. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=10 DB. FAC=1.00.

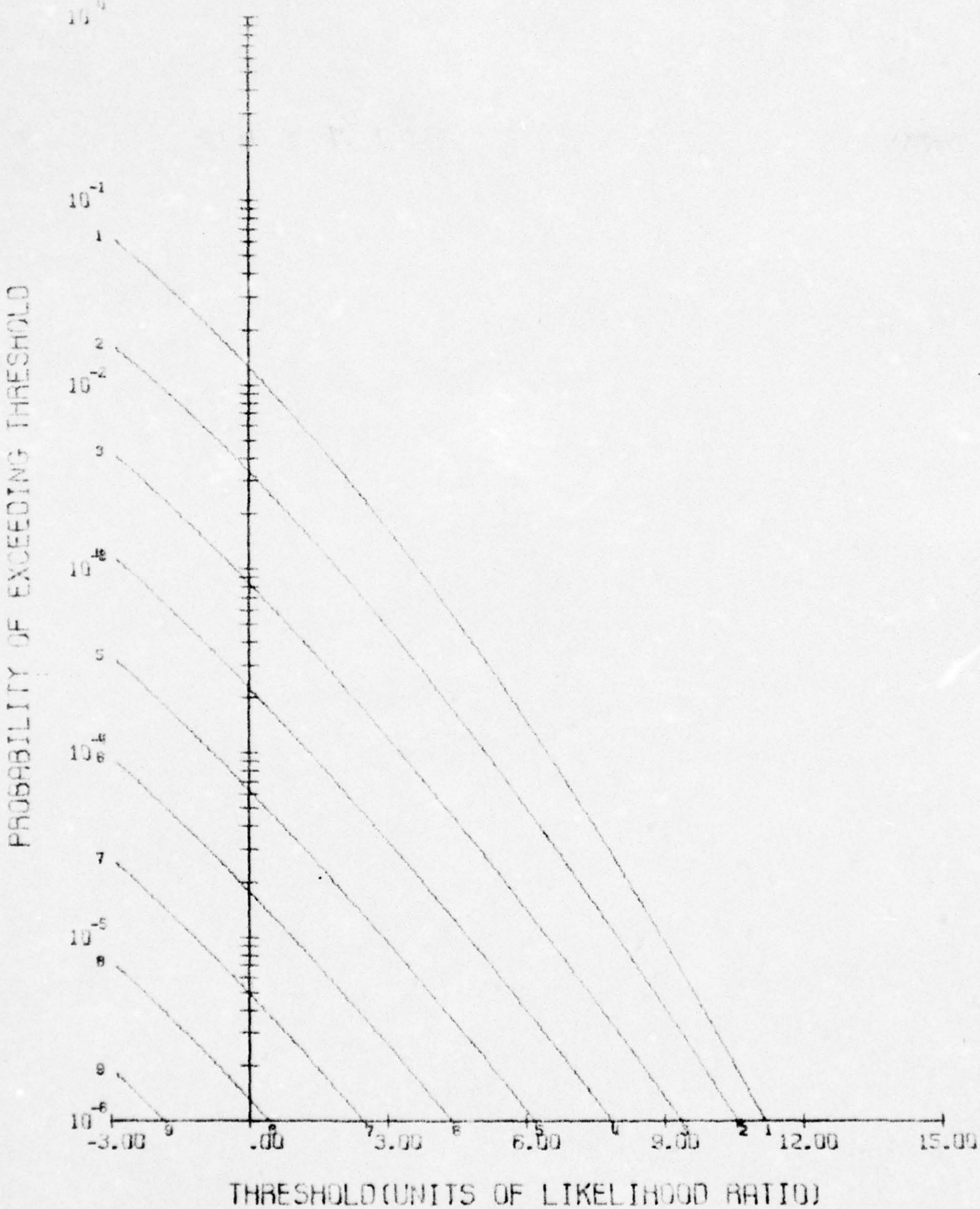

Fig. 16 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR NOISE ONLY. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=14 DB. FAC=1.00.

The results of using a design S/N ratio of 8 dB, the wide tracking windows, and log N_{av} is shown in Fig. 12. Note the rising clutter rate which means that the probability of a noise track crossing any threshold between 3.0 and 15.0 increases with the increasing number of pings the track continues.

Figures 13 through 16 give the results for various values of the design signal-to-noise ratio, using log N and wide multibeam type windows. Note that even for low design signal-to-noise ratios the noise is rejected quickly. This would seem to indicate that one could use low design signal-to-noise ratios.

6. SIGNAL DETECTION

In order to understand the effects on the integration of signal-plus-noise tracks, the analytic tool was applied to distributions for various design signal-to-noise ratios. The results are presented in Figs. 17 through 27. Figure 17 shows the results for a 6 dB target and a design (S/N) of 6 dB. The target distribution is so similar to the maximum noise sample distribution that they cannot be distinguished from one another; hence, the 6 dB target is rejected. For a 10 dB target against a 10 dB design S/N the results are better. There is an integration up at higher thresholds although the probability is about .05 that the track will stop after each ping cycle resulting in lower probability that the track will continue after each ping. In the SLR processor new tracks would be started and, if sufficiently large, old tracks continued, so that the probability of the signal track crossing a threshold after n pings will be higher than shown here. In fact, a lower bound for that probability is given by the upper envelope of the family of curves in the figure. Figures 19 through 23 show the effect of the SLR process on targets with various values of (S/N) for the design (S/N) of 12 dB. In particular, Fig. 21 shows that for track (S/N) of 12 dB the probability that the log likelihood ratio will cross a high threshold becomes very high, a most desirable property. Figure 23 shows the results for the baseline type data. Although this figure shows that the signal-plus-noise track is integrated up faster, the new approach of subtracting log N is superior because the clutter is controlled. Figures 24 through 26 present the results for design S/N of 14 dB.

Finally, Fig. 27 shows the results for a 8 dB target using an 8 dB design signal-to-noise ratio, large windows, and log N_{av}. The probability of detection is increased with increasing ping number for thresholds sufficiently high, but the detection probabilities are fairly low for these values. When compared with Fig. 12 (the noise curves for this set of parameters), the

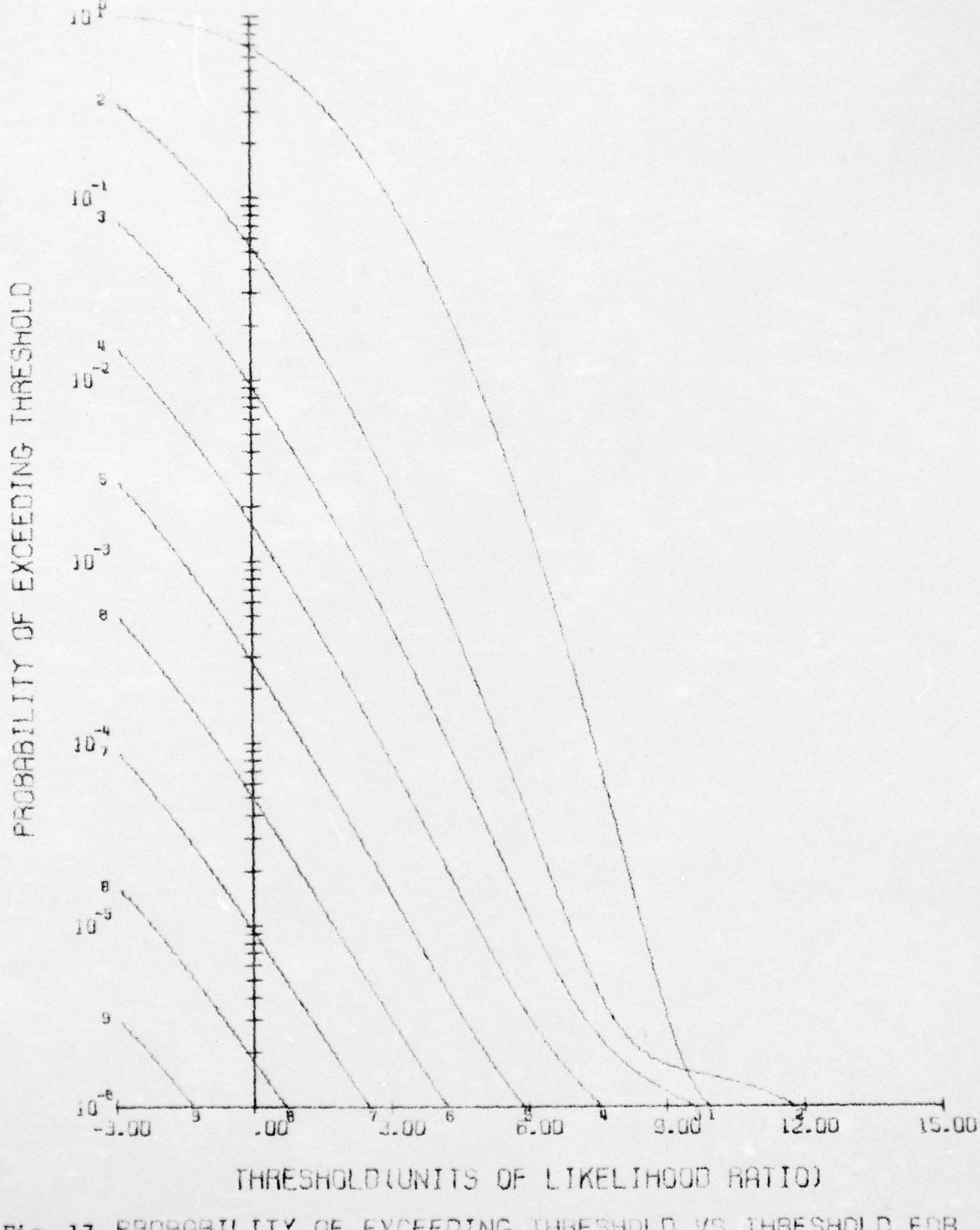

Fig. 17 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR S/N= 6 DB. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N= 6 DB. FAC=1.00.

TRACOR

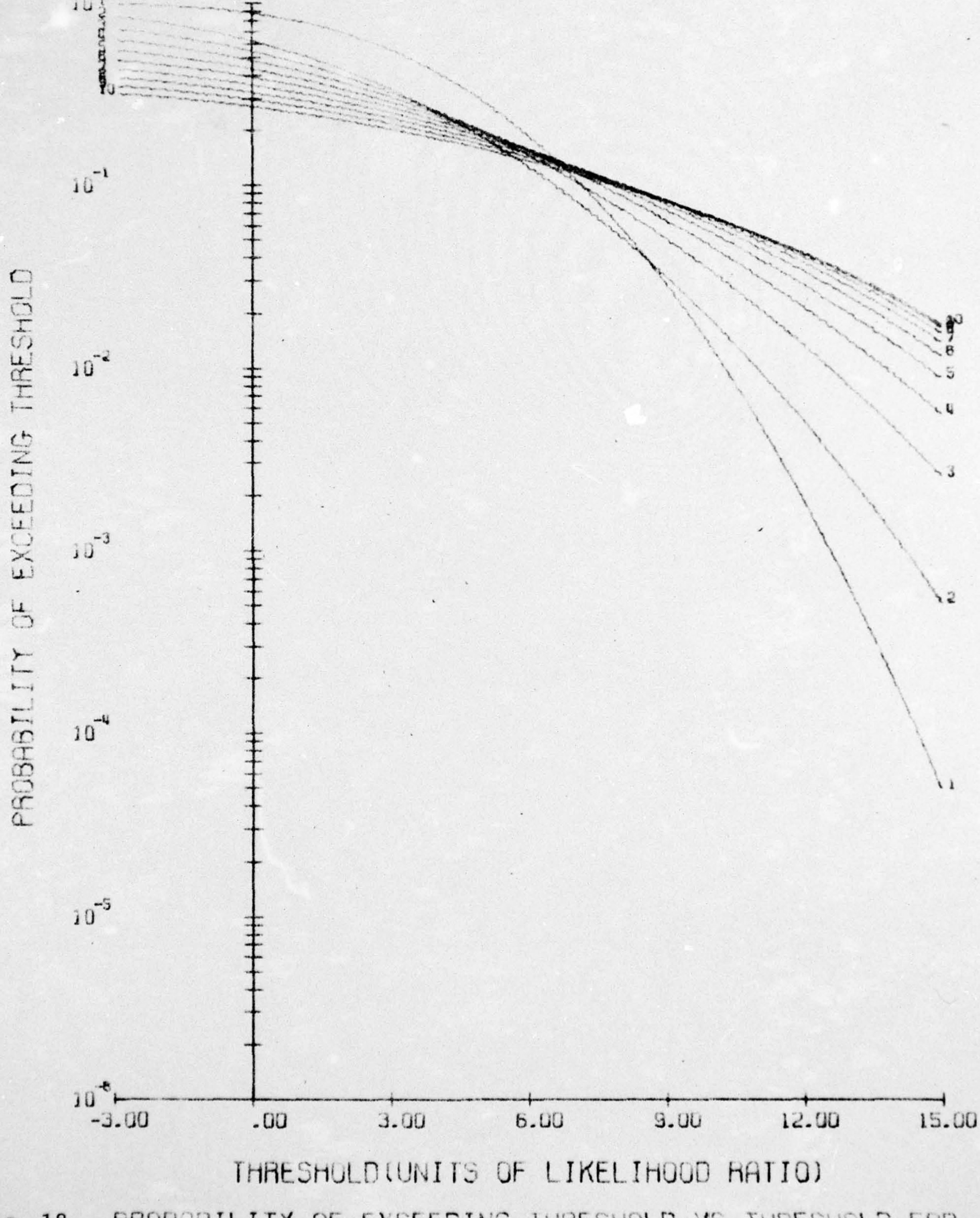

Fig. 18 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR S/N= 10 DB. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=10 DB. FAC=1.00.

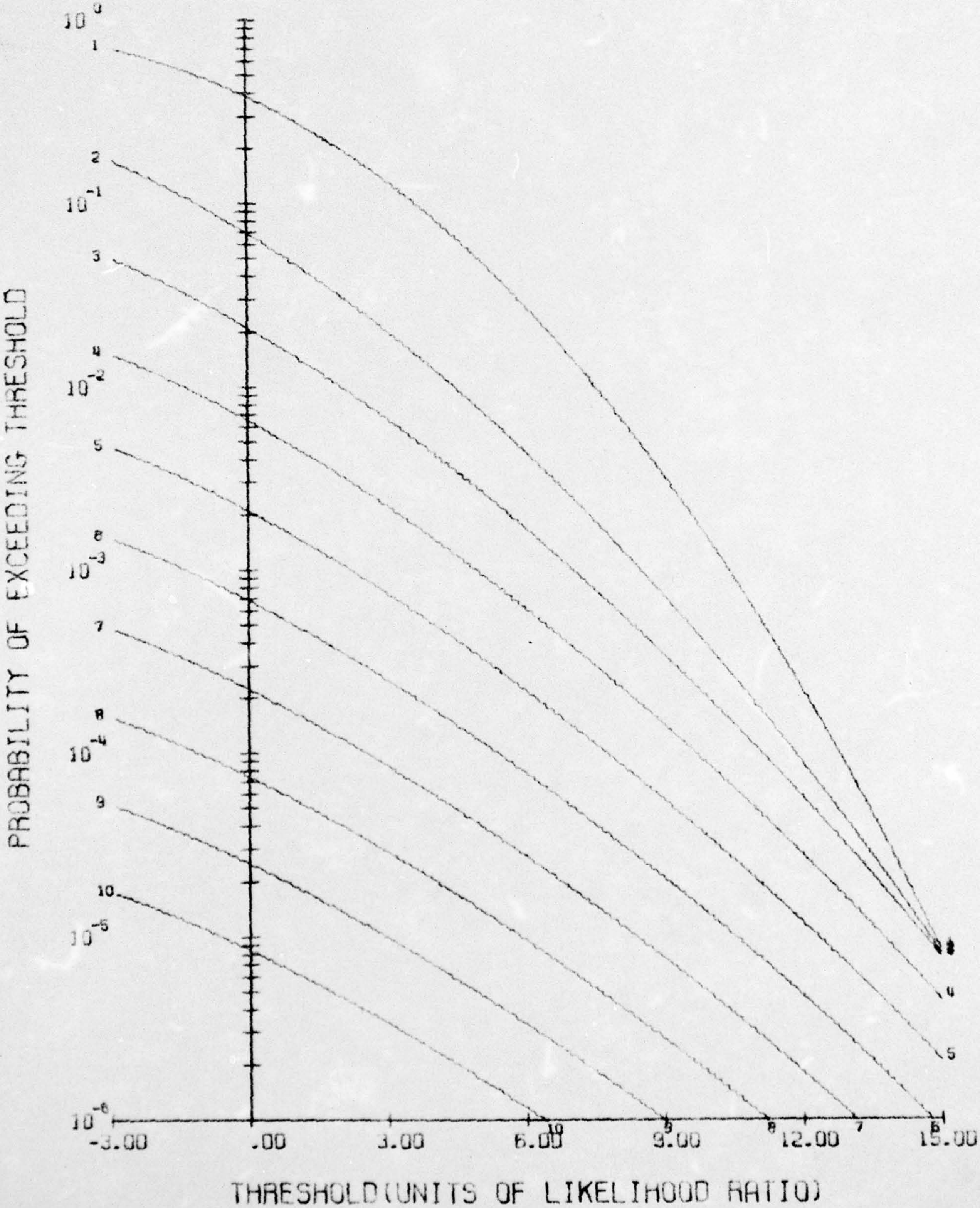

Fig. 19 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR S/N= 6 DB. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=12 DB. FAC=1.00.

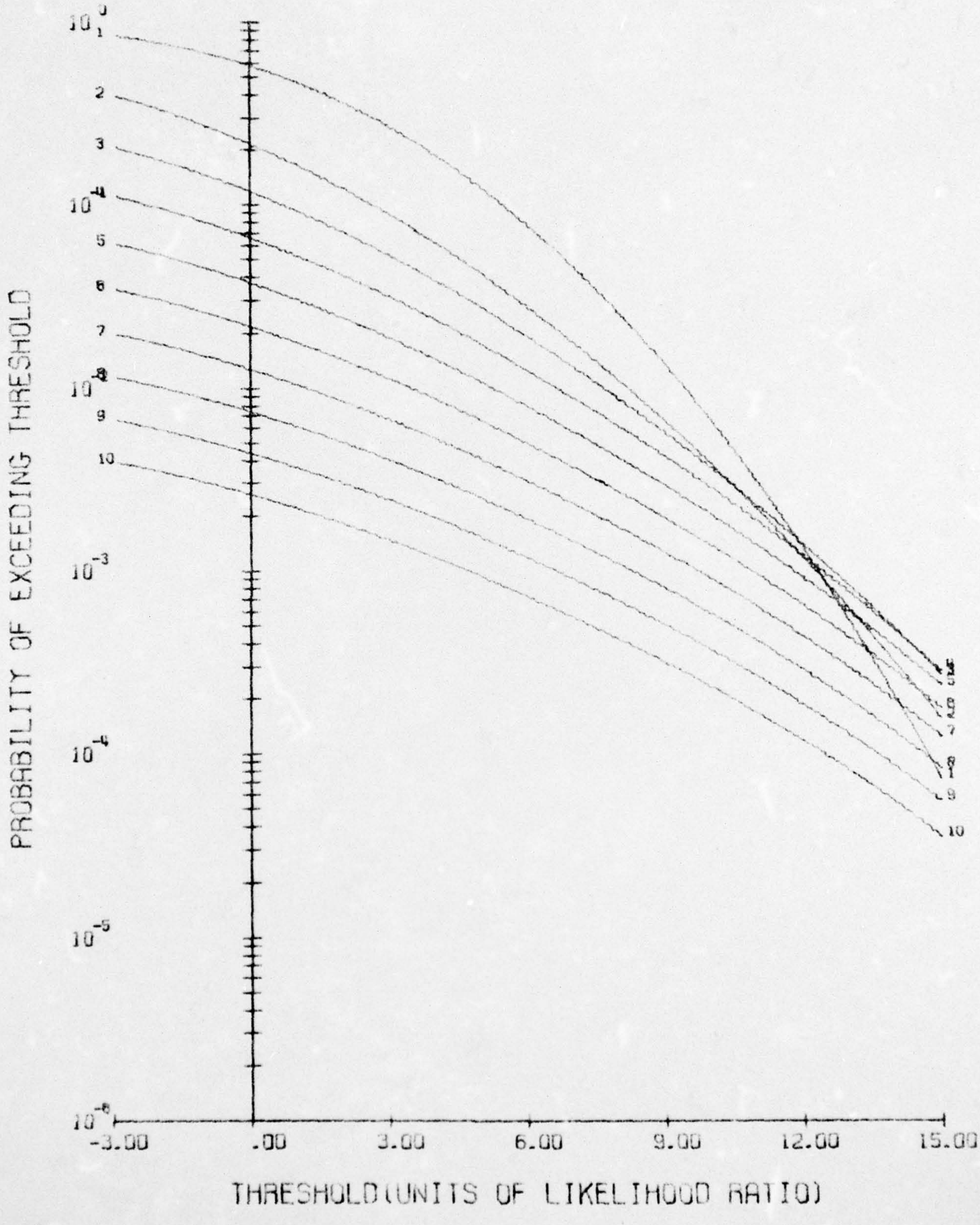

Fig. 20 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR S/N= 8 DB. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=12 DB. FAC=1.00.

Fig. 21 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR S/N= 10 DB. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=12 DB. FAC=1.00.

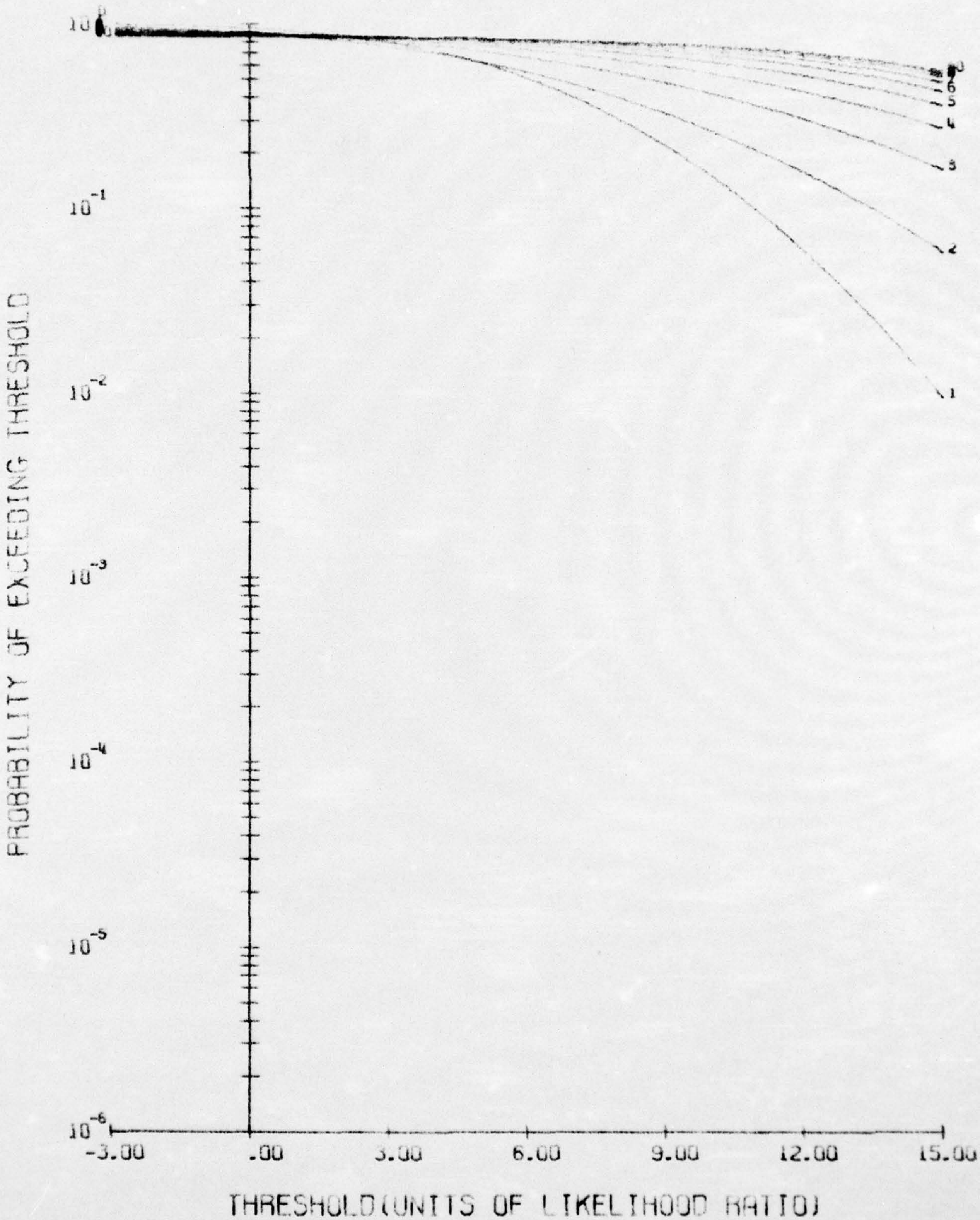

Fig. 22 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR S/N= 12 DB. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=12 DB. FAC=1.00.

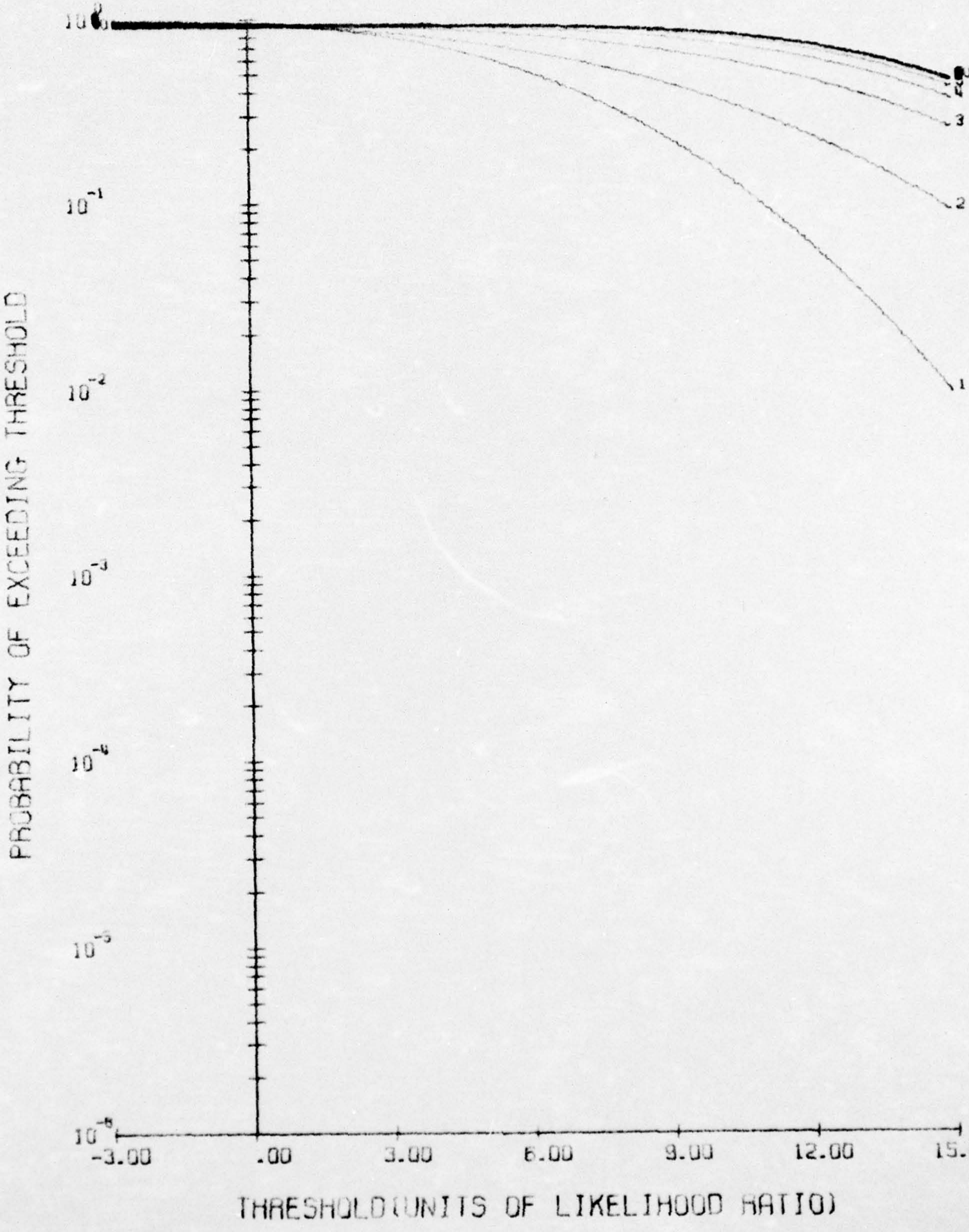

Fig. 23 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR S/N= 12 DB. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=12 DB. FAC= .21.

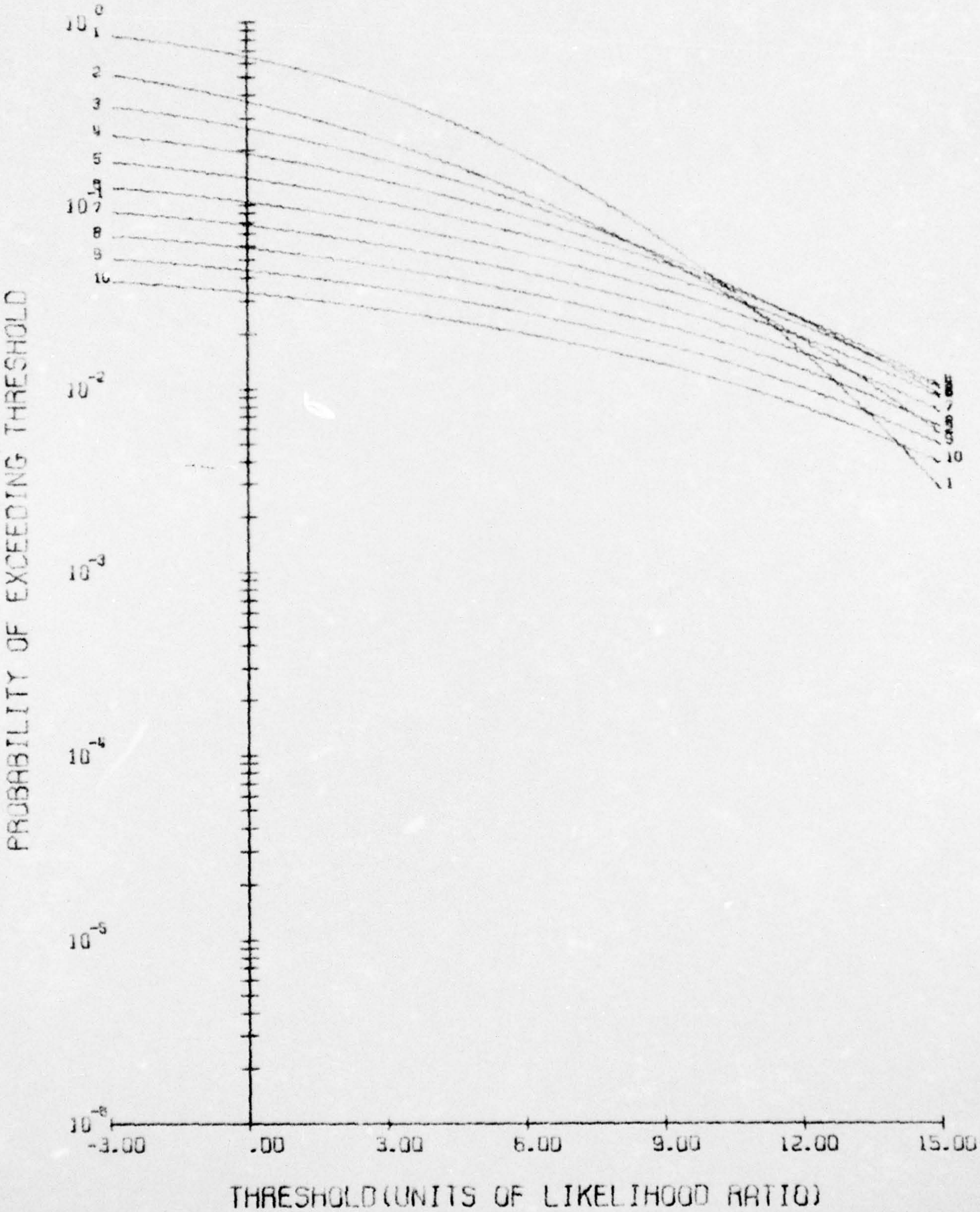

Fig. 24 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR S/N= 10 DB. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=14 DB. FAC=1.00.

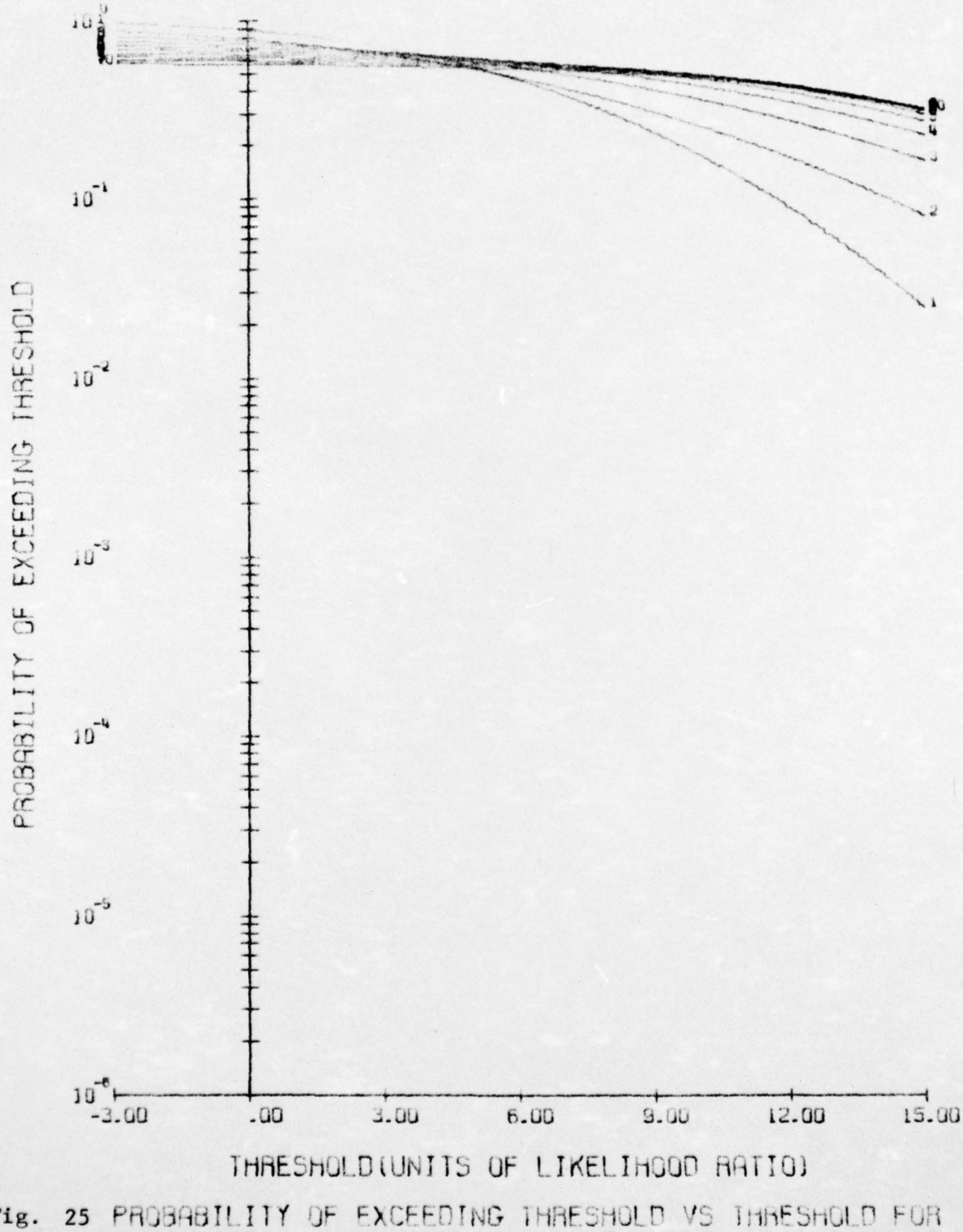

Fig. 25 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR S/N= 12 DB. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=14 DB. FAC=1.00.

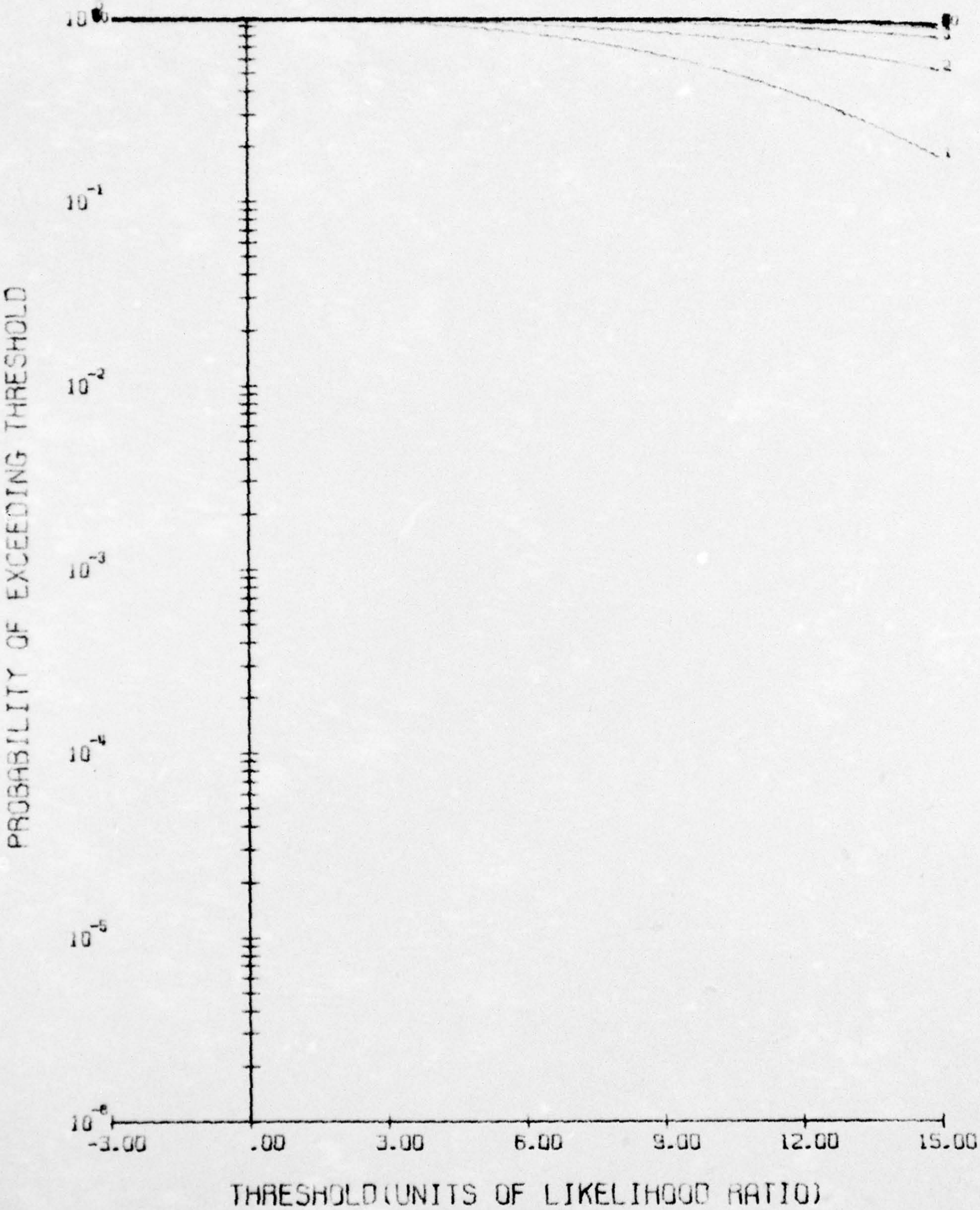

Fig. 26 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR S/N= 14 DB. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N=14 DB. FAC=1.00.

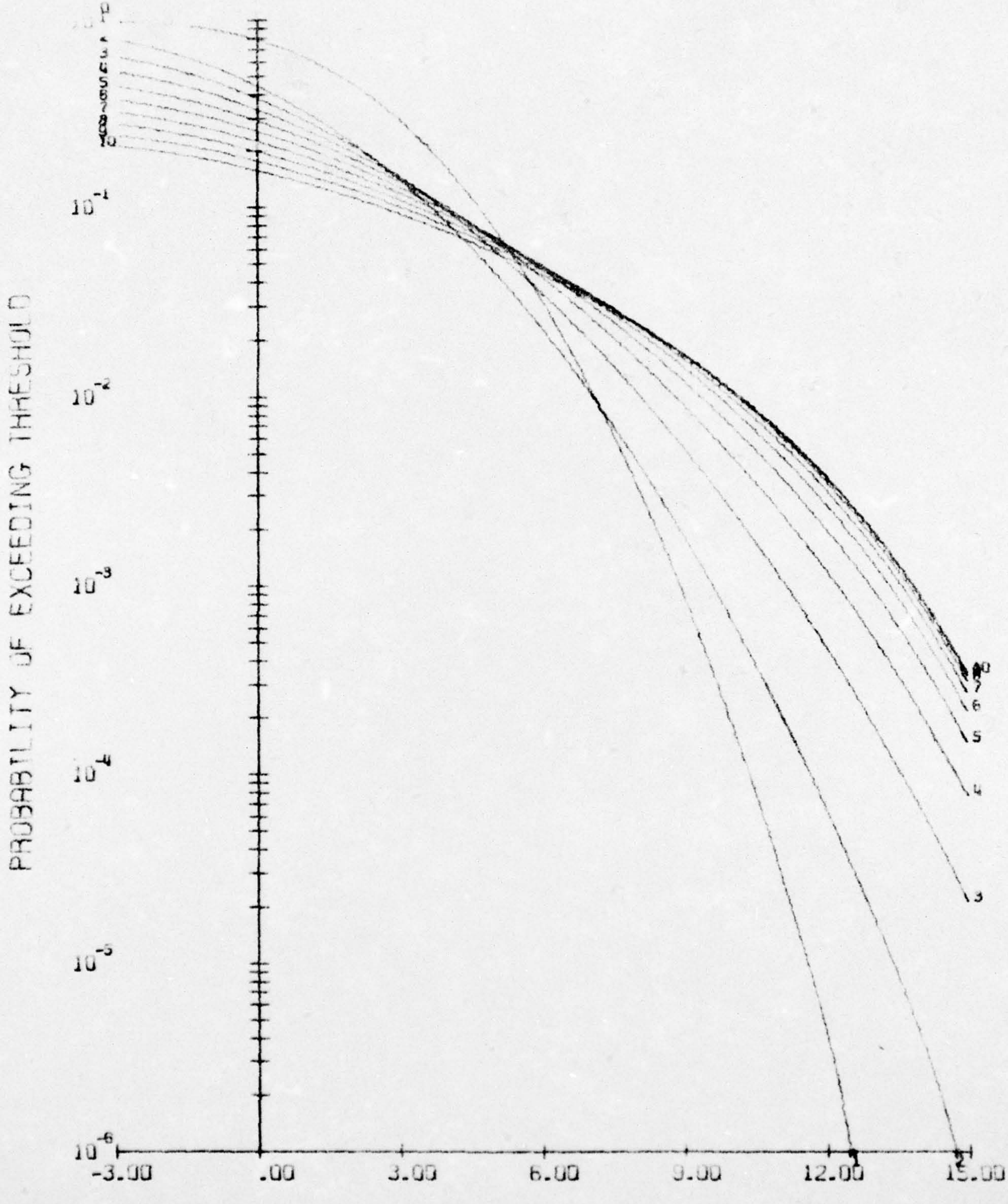

Fig. 27 PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR S/N= 8 DB. NEW TRACKS STARTED ON FIRST PING CYCLE. DESIGN S/N= 8 DB. FAC= .21.

probability of noise clutter is greater than the probability of detection for a significant range of thresholds. Only when the threshold is increased to a higher value does the probability of detection exceed the probability of false alarm. At these thresholds the probability of detection is low, raising a question as to the value of the detector when used in this configuration.

7. SUMMARY

In conclusion the prediction of clutter probability and signal detection indicate the following:

1. Using log N rather than log N_{av} in the SLR processor gives a better control of the clutter probability for lower design signal-to-noise ratios.
2. Tracks with signal-to-noise ratios less than about 8 dB will usually not be detected by the SLR processor regardless of the design S/N, unless additional information is available, e.g. Doppler, or very narrow tracking windows are used.
3. The design S/N of 12 dB still appears to be a good choice for balancing signal detection versus clutter and computer loading.